ARITMETICKÁ POSLOUPNOST

Př 1 Napište prvních 5 členů AP, ve které je dáno:

a) $a_1 = 3$, $d = \frac{1}{2}$

$a_1 = 3$

$a_2 = a_1 + d = 3 + \frac{1}{2} = \frac{6+1}{2} = \frac{7}{2} = 3{,}5$

$a_3 = a_2 + d = 3{,}5 + 0{,}5 = 4$

$a_4 = a_3 + d = 4 + 0{,}5 = 4{,}5$

$a_5 = a_4 + d = 4{,}5 + 0{,}5 = 5$

$3; 3{,}5; 4; 4{,}5; 5$

b) $a_1 = 4$, $d = -2$

$a_1 = 4$

$a_2 = a_1 + d = 4 + (-2) = 4 - 2 = 2$

$a_3 = a_2 + d = 2 + (-2) = 2 - 2 = 0$

$a_4 = a_3 + d = 0 + (-2) = 0 - 2 = -2$

$a_5 = a_4 + d = -2 + (-2) = -2 - 2 = -4$

$4; 2; 0; -2; -4$

c) $a_1 = \frac{-3}{4}$, $d = \frac{-1}{4}$

$a_1 = \frac{-3}{4}$

$a_2 = a_1 + d = \frac{-3}{4} + \left(\frac{-1}{4}\right) = \frac{-3-1}{4} = \frac{-4}{4} = -1$

$a_3 = a_2 + d = -1 + \left(\frac{-1}{4}\right) = \frac{-4-1}{4} = \frac{-5}{4}$

$a_4 = a_3 + d = \frac{-5}{4} + \left(\frac{-1}{4}\right) = \frac{-5-1}{4} = \frac{-6}{4} = \frac{-3}{2}$

$a_5 = a_4 + d = \frac{-6}{4} + \left(\frac{-1}{4}\right) = \frac{-6-1}{4} = \frac{-7}{4}$

$\frac{-3}{4}; -1; \frac{-5}{4}; \frac{-3}{2}; \frac{-7}{4}$

d) $a_1 = 7$, $d = -3$

$a_1 = 7$

$a_2 = a_1 + d = 7 + (-3) = 7 - 3 = 4$

$a_3 = a_2 + d = 4 + (-3) = 4 - 3 = 1$

$a_4 = a_3 + d = 1 + (-3) = 1 - 3 = -2$

$a_5 = a_4 + d = -2 + (-3) = -2 - 3 = -5$

$7; 4; 1; -2; -5$

Př 2 V aritmetické posloupnosti je dáno:

a) $a_4 = 11$, $a_5 = 14$. Vypočtěte a_1, d.

$a_5 = a_4 + d$

$14 = 11 + d$

$d = 14 - 11$

$d = 3$

$a_4 = a_1 + 3d$

$11 = a_1 + 3 \cdot 3$

$11 = a_1 + 9$

$a_1 = 11 - 9$

$a_1 = 2$

$a_1 = 2$, $d = 3$

b) $a_{20} = 35$, $a_{30} = 55$.
Vypočtěte a_{25}.

$a_{25} = a_{20} + 5d$

$a_{30} = a_{25} + 5d$

$a_{30} = a_{20} + 10d$

$55 = 35 + 10d$

$10d = 55 - 35$

$10d = 20$

$d = 2$

$a_{25} = a_{20} + 5d$

$a_{25} = 35 + 5 \cdot 2$

$a_{25} = 35 + 10 = 45$

$a_{25} = 45$

Př 3 V aritmetické posloupnosti je dáno:

a) $a_1 = 7$, $n = 25$, $S_n = 325$. Určete d, a_{25}.

$S_n = \frac{n}{2}(a_1 + a_n)$

$325 = \frac{25}{2}(7 + a_n) \quad /\cdot 2$

$650 = 25(7 + a_n)$

$650 = 175 + 25a_n$

$475 = 25a_n$

$a_n = 19$

$a_n = a_1 + (n-1)d$

$19 = 7 + (25-1)d$

$19 = 7 + 24d$

$26 = 24d$

$d = \frac{26}{24} = \frac{13}{12}$

$d = \frac{13}{12}$, $a_{25} = 19$

b) $S_{14} = 161$, $n = 14$, $d = 1$. Určete a_1, a_{14}.

$S_n = \frac{n}{2}(a_1 + a_n)$

$161 = \frac{14}{2}(a_1 + a_{14})$ ← $a_{14} = a_1 + 13$

$161 = 7 \cdot (a_1 + a_1 + 13)$

$161 = 7 \cdot (2a_1 + 13) \quad /:7$

$23 = 2a_1 + 13$

$-2a_1 = 13 - 23$

$-2a_1 = -10$

$a_1 = 5$

$a_n = a_1 + (n-1)d$

$a_{14} = a_1 + 13$

$a_{14} = 5 + 13$

$a_{14} = 18$

$a_1 = 5$, $a_{14} = 18$

c) $a_n = 97$, $d = 3$, $S_n = 1612$. Určete n, a_1.

$S_n = \frac{n}{2}(a_1 + a_n)$

$1612 = \frac{n}{2}(a_1 + 97)$

$3224 = n(a_1 + 97)$ ← $a_1 = 100 - 3n$

$3224 = n(100 - 3n + 97)$

$3224 = n(197 - 3n)$

$3224 = 197n - 3n^2$

$3n^2 - 197n + 3224 = 0$

$a_n = a_1 + (n-1)d$

$97 = a_1 + (n-1) \cdot 3$

$a_1 = 97 - 3n + 3$

$a_1 = 100 - 3n$

$D = (-197)^2 - 4 \cdot 3 \cdot 3224 =$

$= 38809 - 38688 = 121$

$n_{1,2} = \frac{197 \pm \sqrt{121}}{6} = \frac{197 \pm 11}{6}$

$n_1 = \frac{197 + 11}{6} = \frac{208}{6} = 34,\overline{6} \notin \mathbb{N}$

$n_2 = \frac{197 - 11}{6} = \frac{186}{6} = 31$

$a_1 = 100 - 3n$

$a_1 = 100 - 3 \cdot 31$

$a_1 = 100 - 93$

$a_1 = 7$

$a_1 = 7$, $n = 31$

d) $a_n = 47$, $d = 5$, $S_n = 245$. Určete a_1, n.

$S_n = \frac{n}{2}(a_1 + a_n)$

$245 = \frac{n}{2}(a_1 + 47)$

$490 = n(a_1 + 47)$

$a_n = a_1 + (n-1)d$

$47 = a_1 + (n-1)\cdot 5$

$47 = a_1 + 5n - 5$

$a_1 = 47 - 5n + 5$

$a_1 = 52 - 5n$

$490 = n(52 - 5n + 47)$

$490 = 52n - 5n^2 + 47n$

$5n^2 - 99n + 490 = 0$

$D = (-99)^2 - 4 \cdot 5 \cdot 490 =$

$= 9801 - 9800 = 1$

$n_{1,2} = \frac{99 \pm \sqrt{1}}{10} = \frac{99 \pm 1}{10}$

$n_1 = \frac{99+1}{10} = \frac{100}{10} = \underline{10}$

$n_2 = \frac{99-1}{10} = \frac{98}{10} = 9{,}8 \notin \mathbb{N}$

$a_1 = 52 - 5 \cdot 10 = 52 - 50 = \underline{2}$

$a_1 = 2, n = 10$

Př 4 Určete součet prvních dvanácti členů AP, pro kterou platí:

a) $a_1 = 6$, $a_{12} = 28$

$S_{12} = \frac{12}{2}(a_1 + a_{12})$

$S_{12} = 6 \cdot (6 + 28) = 6 \cdot 34$

$S_{12} = \underline{204}$

$S_{12} = 204$

b) $a_1 = 0$, $d = 1{,}5$

$a_{12} = a_1 + 11d$

$a_{12} = 0 + 11 \cdot 1{,}5$

$a_{12} = \underline{16{,}5}$

$S_{12} = \frac{12}{2}(a_1 + a_{12})$

$S_{12} = 6 \cdot (0 + 16{,}5)$

$S_{12} = 6 \cdot 16{,}5$

$S_{12} = \underline{99}$

$S_{12} = 99$

c) $a_1 = 2$, $a_8 = -19$

$a_8 = a_1 + 7d$

$-19 = 2 + 7d$

$-7d = 2 + 19$

$-7d = 21$

$\underline{d = (-3)}$

$a_{12} = a_1 + 11d$

$a_{12} = 2 + 11 \cdot (-3)$

$a_{12} = 2 - 33$

$a_{12} = \underline{(-31)}$

$S_{12} = \frac{12}{2}(a_1 + a_{12})$

$S_{12} = 6 \cdot (2 - 31)$

$S_{12} = 6 \cdot (-29)$

$S_{12} = \underline{(-174)}$

$S_{12} = (-174)$

d) $a_4 = 7,\ a_8 = -1$

$a_8 = a_4 + (8-4)d$

$-1 = 7 + 4d$

$-4d = 7 + 1$

$-4d = 8$

$d = \underline{(-2)}$

$a_4 = a_1 + 3d$

$7 = a_1 + 3\cdot(-2)$

$7 = a_1 - 6$

$-a_1 = -6 - 7$

$a_1 = \underline{13}$

$a_{12} = a_1 + 11d$

$a_{12} = 13 + 11\cdot(-2)$

$a_{12} = 13 - 22$

$a_{12} = \underline{(-9)}$

$s_{12} = \frac{12}{2}(a_1 + a_{12})$

$s_{12} = 6\cdot(13 - 9)$

$s_{12} = 6\cdot 4$

$s_{12} = \underline{24}$

$\boxed{s_{12} = 24}$

Př 5 Určete aritmetickou posloupnost, ve které platí (tzn. určete a_1, d):

a) $a_2 + a_6 = 32$

$a_4 + a_5 = 36$

$a_1 + d + a_1 + 5d = 32$

$a_1 + 3d + a_1 + 4d = 36$

$2a_1 + 6d = 32 \quad /\cdot(-1)$

$2a_1 + 7d = 36$

$-2a_1 - 6d = -32$

$2a_1 + 7d = 36$

$d = \underline{4}$

$2a_1 + 6d = 32$

$2a_1 + 6\cdot 4 = 32$

$2a_1 + 24 = 32$

$2a_1 = 32 - 24$

$2a_1 = 8$

$a_1 = \underline{4}$

$\boxed{a_1 = 4,\ d = 4}$

b) $a_2 + a_7 = -1$

$a_3 - a_5 = 6$

$a_1 + d + a_1 + 6d = -1$

$a_1 + 2d - (a_1 + 4d) = 6$

$2a_1 + 7d = -1$

$-2d = 6 \ldots -2d = 6$

$d = \underline{(-3)}$

$2a_1 + 7d = -1$

$2a_1 + 7\cdot(-3) = -1$

$2a_1 - 21 = -1$

$2a_1 = 20$

$a_1 = \underline{10}$

$\boxed{a_1 = 10,\ d = (-3)}$

c) $a_1 + a_4 = -14$
$a_2 + a_5 = -10$

$a_1 + a_1 + 3d = -14$
$a_1 + d + a_1 + 4d = -10$

$2a_1 + 3d = -14 \quad /\cdot(-1)$
$2a_1 + 5d = -10$

$-2a_1 - 3d = 14$
$2a_1 + 5d = -10$

$2d = 4$
$d = 2$

$2a_1 + 3d = -14$
$2a_1 + 3\cdot 2 = -14$
$2a_1 + 6 = -14$
$2a_1 = -20$
$a_1 = (-10)$

$a_1 = (-10),\ d = 2$

d) $a_2 + a_5 - a_3 = 27$
$a_1 + a_6 = 52$

$a_1 + d + a_1 + 4d - (a_1 + 2d) = 27$
$a_1 + a_1 + 5d = 52$

$a_1 + 3d = 27 \quad /\cdot(-2)$
$2a_1 + 5d = 52$

$-2a_1 - 6d = -54$
$2a_1 + 5d = 52$

$-d = -2$
$d = 2$

$a_1 + 3d = 27$
$a_1 + 3\cdot 2 = 27$
$a_1 + 6 = 27$
$a_1 = 27 - 6$
$a_1 = 21$

$a_1 = 21,\ d = 2$

Př 6 Určete aritmetickou posloupnost, ve které platí:
$a_1 + a_4 + a_6 = 71$
$a_5 - a_3 - a_2 = 2$

$a_1 + a_1 + 3d + a_1 + 5d = 71$
$a_1 + 4d - (a_1 + 2d) - (a_1 + d) = 2$

$3a_1 + 8d = 71$
$-a_1 + d = 2 \quad /\cdot 3$

$3a_1 + 8d = 71$
$-3a_1 + 3d = 6$

$11d = 77$
$d = 7$

$-a_1 + d = 2$
$-a_1 + 7 = 2$
$-a_1 = -5$
$a_1 = 5$

Kolik členů posloupnosti dává součet 182?

$182 = \frac{n}{2}(5 + a_n)$
$182 = \frac{n}{2}[5 + 5 + (n-1)\cdot 7]$
$364 = n[10 + 7n - 7]$
$364 = n(3 + 7n)$
$364 = 3n + 7n^2$
$7n^2 + 3n - 364 = 0$
$n_{1,2} = \frac{-3 \pm \sqrt{9 + 10192}}{14} = \frac{-3 \pm \sqrt{10201}}{14}$
$n_{1,2} = \frac{-3 \pm 101}{14}$
$n_1 = \frac{-3 + 101}{14} = \frac{98}{14} = 7$
$n_2 = \frac{-3 - 101}{14} = \frac{-104}{14}$ ∉ ℕ

7 členů