

POSLOUPNOSTI - URČENÍ, VLASTNOSTI

Pr 1

Napište prvních šest členů dané posloupnosti:

a) $a_n = \frac{2n+1}{n}$

$$a_1 = \frac{2+1}{1} = \frac{3}{1} = \underline{\underline{3}}$$

$$a_2 = \frac{2 \cdot 2 + 1}{2} = \frac{4+1}{2} = \frac{5}{2} = \underline{\underline{\frac{5}{2}}}$$

$$a_3 = \frac{2 \cdot 3 + 1}{3} = \frac{6+1}{3} = \frac{7}{3} = \underline{\underline{\frac{7}{3}}}$$

$$a_4 = \frac{2 \cdot 4 + 1}{4} = \frac{8+1}{4} = \frac{9}{4} = \underline{\underline{\frac{9}{4}}}$$

$$a_5 = \frac{2 \cdot 5 + 1}{5} = \frac{10+1}{5} = \frac{11}{5} = \underline{\underline{\frac{11}{5}}}$$

$$a_6 = \frac{2 \cdot 6 + 1}{6} = \frac{12+1}{6} = \frac{13}{6} = \underline{\underline{\frac{13}{6}}}$$

$$a_1 = 3, a_2 = \frac{5}{2}, a_3 = \frac{7}{3}, a_4 = \frac{9}{4}, a_5 = \frac{11}{5}, a_6 = \frac{13}{6}$$

b) $a_n = \frac{1-2n}{n+2}$

$$a_1 = \frac{1-2}{1+2} = \frac{-1}{3} = \underline{\underline{-\frac{1}{3}}}$$

$$a_2 = \frac{1-2 \cdot 2}{2+2} = \frac{1-4}{4} = \frac{-3}{4} = \underline{\underline{-\frac{3}{4}}}$$

$$a_3 = \frac{1-2 \cdot 3}{3+2} = \frac{1-6}{5} = \frac{-5}{5} = \underline{\underline{-1}}$$

$$a_4 = \frac{1-2 \cdot 4}{4+2} = \frac{1-8}{6} = \frac{-7}{6} = \underline{\underline{-\frac{7}{6}}}$$

$$a_5 = \frac{1-2 \cdot 5}{5+2} = \frac{1-10}{7} = \frac{-9}{7} = \underline{\underline{-\frac{9}{7}}}$$

$$a_6 = \frac{1-2 \cdot 6}{6+2} = \frac{1-12}{8} = \frac{-11}{8} = \underline{\underline{-\frac{11}{8}}}$$

$$a_1 = -\frac{1}{3}, a_2 = -\frac{3}{4}; a_3 = -1, a_4 = -\frac{7}{6}; a_5 = -\frac{9}{7}; a_6 = -\frac{11}{8}$$

c) $a_n = (-1)^n \cdot \frac{n}{n^2+1}$

$$a_1 = (-1)^1 \cdot \frac{1}{1+1} = -1 \cdot \frac{1}{2} = \underline{\underline{-\frac{1}{2}}}$$

$$a_2 = (-1)^2 \cdot \frac{2}{4+1} = 1 \cdot \frac{2}{5} = \underline{\underline{\frac{2}{5}}}$$

$$a_3 = (-1)^3 \cdot \frac{3}{9+1} = -1 \cdot \frac{3}{10} = \underline{\underline{-\frac{3}{10}}}$$

$$a_4 = (-1)^4 \cdot \frac{4}{16+1} = 1 \cdot \frac{4}{17} = \underline{\underline{\frac{4}{17}}}$$

$$a_5 = (-1)^5 \cdot \frac{5}{25+1} = -1 \cdot \frac{5}{26} = \underline{\underline{-\frac{5}{26}}}$$

$$a_6 = (-1)^6 \cdot \frac{6}{36+1} = 1 \cdot \frac{6}{37} = \underline{\underline{\frac{6}{37}}}$$

$$a_1 = -\frac{1}{2}$$

$$a_2 = \frac{2}{5}$$

$$a_3 = -\frac{3}{10}$$

$$a_4 = \frac{4}{17}$$

$$a_5 = -\frac{5}{26}$$

$$a_6 = \frac{6}{37}$$

$$d) a_n = (n-1) \cdot n$$

$$a_1 = (1-1) \cdot 1 = 0 \cdot 1 = \underline{0}$$

$$a_2 = (2-1) \cdot 2 = 1 \cdot 2 = \underline{2}$$

$$a_3 = (3-1) \cdot 3 = 2 \cdot 3 = \underline{6}$$

$$a_4 = (4-1) \cdot 4 = 3 \cdot 4 = \underline{12}$$

$$a_5 = (5-1) \cdot 5 = 4 \cdot 5 = \underline{20}$$

$$a_6 = (6-1) \cdot 6 = 5 \cdot 6 = \underline{30}$$

$$a_1 = 0, a_2 = 2, a_3 = 6, a_4 = 12, a_5 = 20, a_6 = 30$$

Př2

Vyjděte dané posloupnosti vzorcem pro n -tý člen:

a) 2, 4, 6, 8, ...

$$a_1 = 2 \quad 2 \cdot 1$$

$$a_2 = 4 \quad 2 \cdot 2$$

$$a_3 = 6 \quad 2 \cdot 3$$

$$a_4 = 8 \quad 2 \cdot 4$$

$$a_n = 2n$$

b) 1·2, 2·3, 3·4, 4·5, 5·6, ...

$$a_1 = 1 \cdot 2$$

$$a_2 = 2 \cdot 3$$

$$a_3 = 3 \cdot 4$$

$$a_4 = 4 \cdot 5$$

$$a_n = n \cdot (n+1)$$

c) $\frac{1}{2}; \frac{2}{3}; \frac{3}{4}; \frac{4}{5}; \frac{5}{6}; \dots$

$$a_1 = \frac{1}{2}$$

$$a_2 = \frac{2}{3}$$

$$a_3 = \frac{3}{4}$$

$$a_4 = \frac{4}{5}$$

$$a_n = \frac{n}{n+1}$$

d) 1; -1; 1; -1; 1; -1; ...

$$a_1 = 1$$

$$a_2 = (-1)$$

$$a_3 = 1$$

$$a_4 = (-1)$$

$$a_n = (-1)^{n-1}$$

e) 1, 4, 9, 16, 25, ...

$$a_1 = 1$$

$$a_2 = 4 \quad 2^2$$

$$a_3 = 9 \quad 3^2$$

$$a_4 = 16 \quad 4^2$$

$$a_5 = 25 \quad 5^2$$

$$a_n = n^2$$

Př3

Napište prvních šest členů posloupnosti dané rekurentně:

a) $a_{n+1} = a_n + 7, a_1 = 0$

$$a_2 = a_1 + 7 = 0 + 7 = \underline{7}$$

$$a_3 = a_2 + 7 = 7 + 7 = \underline{14}$$

$$a_4 = a_3 + 7 = 14 + 7 = \underline{21}$$

$$a_5 = a_4 + 7 = 21 + 7 = \underline{28}$$

$$a_6 = a_5 + 7 = 28 + 7 = \underline{35}$$

$$0; 7; 14; 21; 28; 35$$

$$b) a_{n+1} = 1 - a_n, \quad a_1 = 1$$

$$a_2 = 1 - a_1 = 1 - 1 = \underline{0}$$

$$a_3 = 1 - a_2 = 1 - 0 = \underline{1}$$

$$a_4 = 1 - a_3 = 1 - 1 = \underline{0}$$

$$a_5 = 1 - a_4 = 1 - 0 = \underline{1}$$

$$a_6 = 1 - a_5 = 1 - 1 = \underline{0}$$

$$\boxed{1; 0; 1; 0; 1; 0}$$

$$c) a_{n+1} = -a_n, \quad a_1 = 2$$

$$a_2 = -a_1 = \underline{-2}$$

$$a_3 = -a_2 = -(-2) = \underline{2}$$

$$a_4 = -a_3 = \underline{-2}$$

$$a_5 = -a_4 = -(-2) = \underline{2}$$

$$a_6 = -a_5 = \underline{-2}$$

$$\boxed{2; -2; 2; -2; 2; -2}$$

$$d) a_{n+2} = a_{n+1} - a_n, \quad a_1 = 1, a_2 = 1$$

$$a_3 = a_2 - a_1 = 1 - 1 = \underline{0}$$

$$a_4 = a_3 - a_2 = 0 - 1 = \underline{-1}$$

$$a_5 = a_4 - a_3 = -1 - 0 = \underline{-1}$$

$$a_6 = a_5 - a_4 = -1 - (-1) = -1 + 1 = \underline{0}$$

$$\boxed{1; 1; 0; -1; -1; 0}$$

$$e) a_{n+2} = 2a_{n+1} - 3a_n, \quad a_1 = 4, a_2 = 2$$

$$a_3 = 2a_2 - 3a_1 = 2 \cdot 2 - 3 \cdot 4 = 4 - 12 = \underline{-8}$$

$$a_4 = 2a_3 - 3a_2 = 2 \cdot (-8) - 3 \cdot 2 = -16 - 6 = \underline{-22}$$

$$a_5 = 2a_4 - 3a_3 = 2 \cdot (-22) - 3 \cdot (-8) = -44 + 24 = \underline{-20}$$

$$a_6 = 2a_5 - 3a_4 = 2 \cdot (-20) - 3 \cdot (-22) = -40 + 66 = \underline{26}$$

$$\boxed{4; 2; -8; -22; -20; 26}$$

Pr4

Rozhodněte, které z následujících posloupností jsou rostoucí či klesající:

$$a) \quad a_n = \frac{n}{n+1} \quad ? \text{ rost. ?} \quad \frac{n}{n+1} < \frac{n+1}{(n+1)+1}$$

$$\frac{n}{n+1} < \frac{n+1}{n+2}$$

$$\frac{n}{n+1} - \frac{n+1}{n+2} < 0$$

$$\frac{n(n+2) - (n+1)(n+1)}{(n+1)(n+2)} < 0$$

$$\frac{n^2+2n - n^2-2n-1}{(n+1)(n+2)} < 0$$

$$\frac{-1}{(n+1)(n+2)} < 0$$

$$\frac{1}{(n+1)(n+2)} > 0$$

roslouci

	-2	-1	
n+1	-	-	+
n+2	-	+	+
	(+)	-	(+)

$$n \in (-\infty, -2) \cup (-1, \infty)$$

$n \in \mathbb{N} \dots \nearrow$ vyhovuje

b) $a_n = n(n-3)$

? roslouci?

$$n(n-3) < (n+1)(n+1-3)$$

$$n(n-3) < (n+1)(n-2)$$

$$n^2-3n < n^2-n-2$$

$$-3n+n < -2$$

$$-2n < -2 \quad /: (-2)$$

$$n > 1$$

ani R, ani kles.

roslouci od $n=2$

-2, -2, 0, 4, ...

c) $a_n = \frac{n+1}{n}$

? rost.?

$$\frac{n+1}{n} < \frac{n+1+1}{n+1}$$

$$\frac{n+1}{n} - \frac{n+2}{n+1} < 0$$

$$\frac{(n+1)(n+1) - n(n+2)}{n(n+1)} < 0$$

$$\frac{n^2+2n+1 - n^2-2n}{n(n+1)} < 0$$

klesajici

$$\frac{1}{n(n+1)} < 0$$

	-1	0	
n	-	-	+
n+1	-	+	+
	+	(-)	+

$$n \in (-1, 0) \text{ roste, jinak kles}$$

d) $a_n = \frac{-1}{n^2}$

$$\frac{-1}{n^2} < \frac{-1}{(n+1)^2}$$

? rost.?

$$\frac{-1}{n^2} + \frac{1}{(n+1)^2} < 0$$

$$\frac{-(n+1)^2 + n^2}{n^2 \cdot (n+1)^2} < 0$$

$$\frac{-n^2-2n-1 + n^2}{n^2 \cdot (n+1)^2} < 0$$

$$\frac{-2n-1}{n^2 \cdot (n+1)^2} < 0$$

poz. (vždy +) , +, -2n-1 < 0
-2n-1 < 0
-2n < 1
n > -1/2

$$\frac{-2n-1}{n^2(n+1)^2} < 0$$

	-1	-1/2	0	
-2n-1	+	+	-	-
n^2	+	+	+	+
(n+1)^2	+	+	+	+
	+	+	(-)	(-)

$$n \in (-1/2, 0) \cup (0, \infty)$$

roslouci