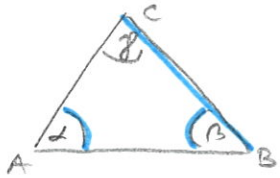
SINOVÁ VĚTA

Př 1) Určete délky zbývajících stran a velikosti zbývajících vnitřních úhlů trojúhelníku ABC, je-li dáno:

a) $a = 20\ cm$, $\alpha = 45°$, $\beta = 30°$



γ: $\gamma = 180° - (\alpha + \beta)$

$\gamma = 180° - (45° + 30°)$

$\gamma = 180° - 75°$

$\gamma = 105°$

b: $\frac{b}{\sin\beta} = \frac{a}{\sin\alpha}$

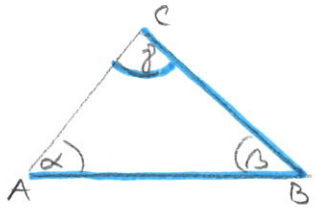
$b = \frac{a \cdot \sin\beta}{\sin\alpha} = \frac{20 \cdot \sin 30°}{\sin 45°} = \frac{20 \cdot \frac{1}{2}}{\frac{\sqrt{2}}{2}} = \frac{10}{\frac{\sqrt{2}}{2}} = 10 \cdot \frac{2}{\sqrt{2}} = \frac{20}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} =$

$= \frac{20\sqrt{2}}{2} = 10\sqrt{2} \doteq 14{,}14\ cm$

c: $\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha}$

$c = \frac{a \cdot \sin\gamma}{\sin\alpha} = \frac{20 \cdot \sin 105°}{\sin 45°} = \frac{20 \cdot 0{,}9659}{0{,}7071} \doteq 27{,}32\ cm$

b) $a = 8{,}6\ mm$, $c = 11{,}4\ mm$, $\gamma = 74°20'$



α: $\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$

$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$

$\sin\alpha = \frac{\sin\gamma \cdot a}{c} = \frac{\sin 74°20' \cdot 8{,}6}{11{,}4} = \frac{0{,}9628 \cdot 8{,}6}{11{,}4} \doteq 0{,}7263$

$\alpha \doteq 46°35'$

β: $\beta = 180° - (\alpha + \gamma)$

$\beta = 180° - (46°35' + 74°20') = 180° - 120°55'$

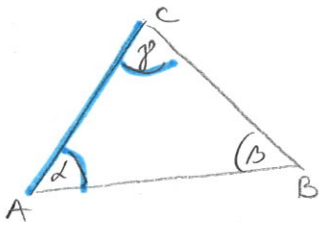
$\beta = 59°5'$

b: $\frac{b}{\sin\beta} = \frac{a}{\sin\alpha}$

$b = \frac{a \cdot \sin\beta}{\sin\alpha} = \frac{8{,}6 \cdot \sin 59°5'}{\sin 46°35'} = \frac{8{,}6 \cdot 0{,}8579}{0{,}7263}$

$b \doteq 10{,}2\ mm$

c) $b = 48{,}55$ cm, $\gamma = 6°30'$, $\alpha = 69°40'$



β: $\beta = 180° - (\alpha + \gamma)$

$\beta = 180° - (69°40' + 6°30')$

$\beta = 180° - 76°10'$

$\beta = 103°50'$

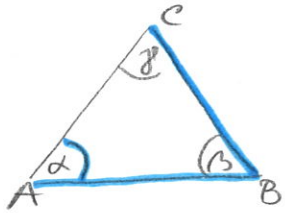
a: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$

$a = \frac{b \cdot \sin\alpha}{\sin\beta} = \frac{b \cdot \sin 69°40'}{\sin 103°50'} = \frac{48{,}55 \cdot 0{,}9376}{0{,}9709} \doteq 46{,}88$ cm

c: $\frac{c}{\sin\gamma} = \frac{b}{\sin\beta}$

$c = \frac{b \cdot \sin\gamma}{\sin\beta} = \frac{b \cdot \sin 6°30'}{\sin 103°50'} = \frac{48{,}55 \cdot 0{,}1132}{0{,}9709} \doteq 5{,}66$ cm

d) $a = 140$ mm, $c = 300$ mm, $\alpha = 71°14'$



γ: $\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha}$

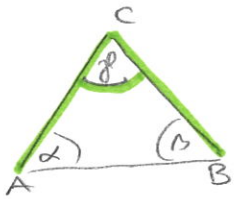
$\frac{\sin\gamma}{c} = \frac{\sin\alpha}{a}$

$\sin\gamma = \frac{\sin\alpha \cdot c}{a} = \frac{\sin 71°14' \cdot 300}{140} = \frac{0{,}9468 \cdot 300}{140} = 2{,}0289$

úloha nemá řešení
(špatně použitá sinova věta – neznáme úhel proti větší straně)

Př 2) Určete obsah trojúhelníku ABC, je-li dáno:

a) $a = 6{,}4$ dm, $b = 4{,}7$ dm, $\gamma = 68°$



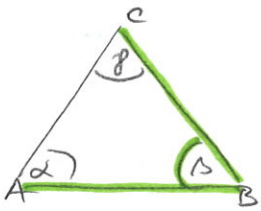
$S = \frac{1}{2} ab \cdot \sin\gamma$

$S = \frac{1}{2} \cdot 6{,}4 \cdot 4{,}7 \cdot \sin 68°$

$S = \frac{1}{2} \cdot 6{,}4 \cdot 4{,}7 \cdot 0{,}9271$

$S \doteq 13{,}94$ dm^2

b) $a = 7{,}5$ cm, $c = 9{,}2$ cm, $\beta = 134°$



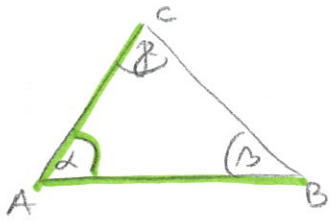
$S = \frac{1}{2} ac \sin\beta$

$S = \frac{1}{2} \cdot 7{,}5 \cdot 9{,}2 \cdot \sin 134°$

$S = \frac{1}{2} \cdot 7{,}5 \cdot 9{,}2 \cdot 0{,}7193$

$S \doteq 24{,}82$ cm^2

c) $b = 72{,}5$ mm, $c = 56{,}7$ mm, $\alpha = 74°12'$



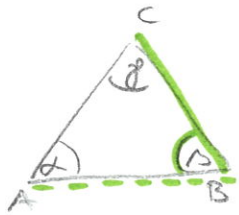
$$S = \frac{1}{2} bc \sin\alpha$$

$$S = \frac{1}{2} \cdot 72{,}5 \cdot 56{,}7 \cdot \sin 74°12'$$

$$S = \frac{1}{2} \cdot 72{,}5 \cdot 56{,}7 \cdot 0{,}9622$$

$$S \doteq 1977{,}68 \text{ mm}^2$$

Př3) Obsah trojúhelníku ABC je $64{,}6\ m^2$, $a = 9{,}4$ m, $\beta = 75°$. Vypočítejte velikost strany c.



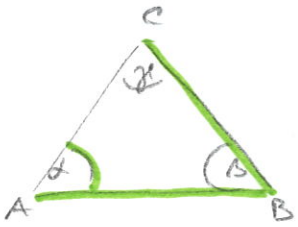
$$S = \frac{1}{2} ac \sin\beta$$

$$\frac{1}{2} ac \sin\beta = S \quad / \cdot \frac{2}{a \sin\beta}$$

$$c = \frac{2S}{a \sin\beta} = \frac{2 \cdot 64{,}6}{9{,}4 \cdot \sin 75°} = \frac{129{,}2}{9{,}4 \cdot 0{,}9659}$$

$$c \doteq 14{,}23 \text{ m}$$

Př4) Určete délky všech stran a velikosti všech vnitřních úhlů trojúhelníku ABC, je-li dáno $a = 11{,}6$ m, $c = 9$ m, $\alpha = 65°30'$. Dále určete obsah $\triangle ABC$.



γ: $\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$

$$\sin\gamma = \frac{c \cdot \sin\alpha}{a} = \frac{9 \cdot \sin 65°30'}{11{,}6}$$

$$\sin\gamma = \frac{9 \cdot 0{,}9099}{11{,}6} = 0{,}7059 \qquad \gamma = 44°54'$$

β: $\beta = 180° - (\alpha + \gamma) = 180° - (65°30' + 44°54') = 180° - 110°24'$

$\beta = 69°36'$

b: $\frac{b}{\sin\beta} = \frac{a}{\sin\alpha}$ $\quad b = \frac{a \cdot \sin\beta}{\sin\alpha} = \frac{11{,}6 \cdot \sin 69°36'}{\sin 65°30'} = \frac{11{,}6 \cdot 0{,}9372}{0{,}9099}$

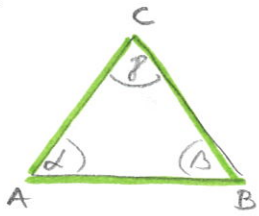
$b = 11{,}95$ m

S: $S = \frac{1}{2} ac \sin\beta = \frac{1}{2} \cdot 11{,}6 \cdot 9 \cdot \sin 69°36'$

$S = \frac{1}{2} \cdot 11{,}6 \cdot 9 \cdot 0{,}9372 = 48{,}92\ m^2$

KOSINOVÁ VĚTA

Př 1) Určete délky zbývajících stran a velikosti zbývajících vnitřních úhlů trojúhelníku ABC, je-li dáno:



a) $a = 16{,}9$ mm, $b = 21{,}8$ mm, $c = 19{,}4$ mm.

α: $a^2 = b^2 + c^2 - 2bc\cdot\cos\alpha$

$16{,}9^2 = 21{,}8^2 + 19{,}4^2 - 2\cdot 21{,}8\cdot 19{,}4\cdot\cos\alpha$

$285{,}61 = 475{,}24 + 376{,}36 - 845{,}84\cdot\cos\alpha$

$845{,}84\cos\alpha = 851{,}6 - 285{,}61$

$845{,}84\cos\alpha = 565{,}99$

$\cos\alpha = 0{,}6691$

$\alpha \doteq 48°$

β: $b^2 = a^2 + c^2 - 2ac\cos\beta$

$21{,}8^2 = 16{,}9^2 + 19{,}4^2 - 2\cdot 16{,}9\cdot 19{,}4\cdot\cos\beta$

$475{,}24 = 285{,}61 + 376{,}36 - 655{,}72\cos\beta$

$655{,}72\cos\beta = 661{,}97 - 475{,}24$

$655{,}72\cos\beta = 186{,}73$

$\cos\beta = 0{,}2847$

$\beta \doteq 73°27'$

γ:

$\gamma = 180° - (\alpha + \beta)$

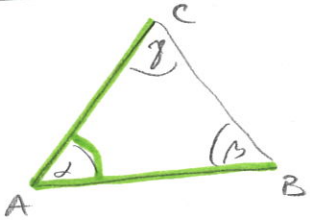
$\gamma = 180° - (48° + 73°27')$

$\gamma = 180° - 121°27'$

$\gamma = 58°33'$

(pozn. pokud nejprve spočítáme α, musíme na další úhel použít použe kosinovou větu; sinová nesplňuje podmínky užití)

b) $b = 64{,}1$ cm, $c = 29{,}3$ cm, $\alpha = 48°20'$



a: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

$a^2 = 64{,}1^2 + 29{,}3^2 - 2\cdot 64{,}1\cdot 29{,}3\cdot\cos 48°20'$

$a^2 = 4108{,}81 + 858{,}49 - 3756{,}26\cdot 0{,}6647$

$a^2 \doteq 4967{,}3 - 2496{,}79$

$a^2 = 2470{,}51$

$a \doteq 49{,}7$ cm

γ: $\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha}$ (lze použít – známe úhel proti větší straně)

$\frac{\sin\gamma}{c} = \frac{\sin\alpha}{a}$ pozn. β by nešlo

$\sin\gamma = \frac{\sin\alpha\cdot c}{a}$

$\sin\gamma = \frac{\sin 48°20'\cdot 29{,}3}{49{,}7} = \frac{0{,}7470\cdot 29{,}3}{49{,}7}$

$\sin\gamma = 0{,}4403$

$\gamma = 26°7'$

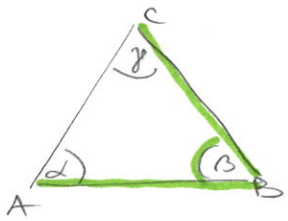
β: $\beta = 180° - (\alpha + \gamma)$

$\beta = 180° - (48°20' + 26°7')$

$\beta = 180° - 74°27'$

$\beta = 105°33'$

c) $a = 0{,}15\ m$, $c = 0{,}27 m$, $\beta = 110°59'$



b: $b^2 = a^2 + c^2 - 2ac\cos\beta$

$b^2 = 0{,}15^2 + 0{,}27^2 - 2 \cdot 0{,}15 \cdot 0{,}27 \cdot \cos 110°59'$

$b^2 = 0{,}0225 + 0{,}0729 - 0{,}081 \cdot (-0{,}3580)$

$b^2 = 0{,}0954 + 0{,}028998$

$b^2 = 0{,}124398$

$b \doteq 0{,}35\ m$

α: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$

$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$

$\sin\alpha = \frac{\sin\beta \cdot a}{b} = \frac{\sin 110°59' \cdot 0{,}15}{0{,}35} = \frac{0{,}9336 \cdot 0{,}15}{0{,}35}$

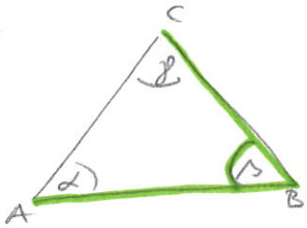
$\sin\alpha = 0{,}4001 \qquad \alpha \doteq 23°35'$

γ: $\gamma = 180° - (\alpha + \beta)$

$\gamma = 180° - (23°35' + 110°59') = 180° - 134°34'$

$\gamma = 45°26'$

d) $a = 51{,}32\ dm$, $c = 34{,}76\ dm$, $\beta = 126°12'$



b: $b^2 = a^2 + c^2 - 2ac\cos\beta$

$b^2 = 51{,}32^2 + 34{,}76^2 - 2 \cdot 51{,}32 \cdot 34{,}76 \cdot \cos 126°12'$

$b^2 = 2633{,}7424 + 1208{,}2576 - 3567{,}7664 \cdot (-0{,}5906)$

$b^2 = 3842 + 2107{,}122836$

$b^2 = 5949{,}122836$

$b \doteq 77{,}13\ dm$

c: $\frac{c}{\sin\gamma} = \frac{b}{\sin\beta}$

$\frac{\sin\gamma}{c} = \frac{\sin\beta}{b}$

$\sin\gamma = \frac{\sin\beta \cdot c}{b} = \frac{\sin 126°12' \cdot 34{,}76}{77{,}13}$

$\sin\gamma = \frac{0{,}8069 \cdot 34{,}76}{77{,}13} = 0{,}3636$

$\gamma = 21°19'$

α: $\alpha = 180° - (\beta + \gamma)$

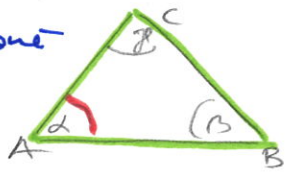
$\alpha = 180° - (126°12' + 21°19')$

$\alpha = 180° - 147°31'$

$\alpha = 32°29'$

Př2) Vypočtěte velikost největšího vnitřního úhlu trojúhelníku ABC v němž je $a = 74\,m$, $b = 53\,m$, $c = 45\,m$.

největší vnitřní úhel – leží proti největší straně



α: $a^2 = b^2 + c^2 - 2bc \cdot \cos\alpha$

$74^2 = 53^2 + 45^2 - 2 \cdot 53 \cdot 45 \cdot \cos\alpha$

$5476 = 2809 + 2025 - 4770 \cdot \cos\alpha$

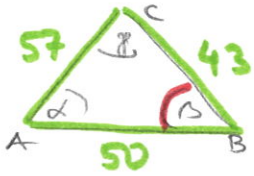
$4770 \cos\alpha = 4834 - 5476$

$4770 \cos\alpha = -642 \quad / : 4770$

$\cos\alpha = -0{,}1345$

$\alpha \doteq 97°44'$

Př3) Vypočtěte velikost největšího vnitřního úhlu trojúhelníku ABC jehož strany mají délky 43mm, 57mm, 50mm.



β: $b^2 = a^2 + c^2 - 2ac \cdot \cos\beta$

$57^2 = 43^2 + 50^2 - 2 \cdot 43 \cdot 50 \cos\beta$

$3249 = 1849 + 2500 - 4300 \cos\beta$

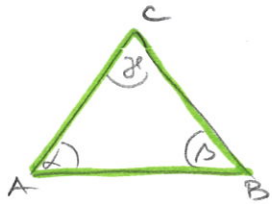
$4300 \cos\beta = 4349 - 3249$

$4300 \cos\beta = 1100 \quad / : 4300$

$\cos\beta = 0{,}2558$

$\beta \doteq 75°11'$

Př4) Určete délky všech stran a velikosti všech vnitřních úhlů trojúhelníku ABC, je-li dáno $a = 16{,}9\,cm$, $b = 26\,cm$, $c = 27{,}3\,cm$. Dále určete obsah $\triangle ABC$.



γ: $c^2 = a^2 + b^2 - 2ab\cos\gamma$

$27{,}3^2 = 16{,}9^2 + 26^2 - 2 \cdot 16{,}9 \cdot 26 \cdot \cos\gamma$

$745{,}29 = 285{,}61 + 676 - 878{,}8\cos\gamma$

$878{,}8\cos\gamma = 961{,}61 - 745{,}29$

$878{,}8\cos\gamma = 216{,}32$

$\cos\gamma \doteq 0{,}2461$

$\gamma \doteq 75°45'$

α: $\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$

$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$

$\sin\alpha = \frac{\sin\gamma \cdot a}{c} = \frac{\sin 75°45' \cdot 16{,}9}{27{,}3}$

$\sin\alpha = \frac{0{,}9692 \cdot 16{,}9}{27{,}3} = 0{,}5999$

$\alpha \doteq 36°52'$

β: $\beta = 180° - (\alpha + \gamma)$

$\beta = 180° - (36°52' + 75°45')$

$\beta = 180° - 112°37'$

$\beta = 67°23'$

$S = \frac{1}{2}ab\sin\gamma = \frac{1}{2} \cdot 16{,}9 \cdot 26 \cdot \sin 75°45' =$

$= \frac{1}{2} \cdot 16{,}9 \cdot 26 \cdot 0{,}9692$

$S \doteq 212{,}93\,cm^2$