GONIOMETRICKÉ VZORCE – ÚPRAVY VÝRAZŮ

Př 1 Zjednodušte dané výrazy:

a) $\left(\frac{1}{\cos x}+\operatorname{tg}x\right)\cdot\left(\frac{1}{\cos x}-\operatorname{tg}x\right)=$

$=\left(\frac{1}{\cos x}+\frac{\sin x}{\cos x}\right)\cdot\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)=\frac{1+\sin x}{\cos x}\cdot\frac{1-\sin x}{\cos x}=$

$=\frac{1-\sin^2 x}{\cos^2 x}=\frac{\cos^2 x}{\cos^2 x}=\underline{1}$ podm. $\cos x\neq 0$

b) $1-\sin^2 x+\operatorname{cotg}^2 x\cdot\sin^2 x=\underbrace{1-\sin^2 x}+\frac{\cos^2 x}{\cancel{\sin^2 x}}\cdot\cancel{\sin^2 x}=$

$=\cos^2 x+\cos^2 x=\underline{2\cos^2 x}$ podm.: $\sin x\neq 0$

c) $\underbrace{\sin^2 x+\cos^2 x}+\operatorname{cotg}^2 x=1+\frac{\cos^2 x}{\sin^2 x}=\frac{\sin^2 x+\cos^2 x}{\sin^2 x}=\underline{\frac{1}{\sin^2 x}}$

podm.: $\sin x\neq 0$

d) $\operatorname{cotg}x+\frac{\sin x}{1+\cos x}=\frac{\cos x}{\sin x}+\frac{\sin x}{1+\cos x}=$

$=\frac{\cos x(1+\cos x)+\sin^2 x}{\sin x\cdot(1+\cos x)}=\frac{\cos x+\overbrace{\cos^2 x+\sin^2 x}}{\sin x\cdot(1+\cos x)}=$

$=\frac{\cancel{\cos x+1}}{\sin x\,\cancel{(1+\cos x)}}=\underline{\frac{1}{\sin x}}$ podm. $\sin x\neq 0$, $\cos x\neq -1$

Př 2 Zjednodušte:

a) $\frac{\sin x+\cos x}{1+\operatorname{tg}x}=\frac{\sin x+\cos x}{1+\frac{\sin x}{\cos x}}=\frac{\sin x+\cos x}{\frac{\cos x+\sin x}{\cos x}}=\frac{\cancel{\sin x+\cos x}}{1}\cdot\frac{\cos x}{\cancel{\cos x+\sin x}}=$

$=\frac{\cos x}{1}=\underline{\cos x}$ podm. $\operatorname{tg}x\neq -1$ $(\cos x\neq -\sin x)$, $\cos x\neq 0$

b) $\frac{\cos x}{1-\sin x}-\frac{\cos x}{1+\sin x}=\frac{\cos x(1+\sin x)-\cos x(1-\sin x)}{(1-\sin x)(1+\sin x)}=$

$=\frac{\cancel{\cos x}+\sin x\cos x-\cancel{\cos x}+\sin x\cos x}{1-\sin^2 x}=\frac{2\sin x\cancel{\cos x}}{\cos^{\cancel{2}} x}=\underline{\frac{2\sin x}{\cos x}}$

$\sin x\neq\pm 1$ $(\cos x\neq 0)$

c) $\frac{\cos x+\sin x}{\cos x-\sin x}-\frac{\cos x-\sin x}{\cos x+\sin x}=\frac{(\cos x+\sin x)(\cos x+\sin x)-(\cos x-\sin x)(\cos x-\sin x)}{(\cos x-\sin x)(\cos x+\sin x)}$

$=\frac{\cancel{\cos^2 x}+2\sin x\cos x+\cancel{\sin^2 x}-\cancel{\cos^2 x}+2\sin x\cos x-\cancel{\sin^2 x}}{\cos^2 x-\sin^2 x}=\frac{2\cdot\sin 2x}{\cos 2x}=$

$=\underline{2\operatorname{tg}2x}$ $\cos x\neq\pm\sin x$, $\cos 2x\neq 0$

$\boxed{\text{Př 3}}$ Dokažte, že platí:

a) $\dfrac{\cot x - \cos x}{\cot x} = 1 - \sin x$

$$L = \frac{\cot x - \cos x}{\cot x} = \frac{\cancel{\cot x}}{\cancel{\cot x}} - \frac{\cos x}{\cot x} = 1 - \frac{\cos x}{\frac{\cos x}{\sin x}} = 1 - \frac{\cancel{\cos x}}{1}\cdot\frac{\sin x}{\cancel{\cos x}} =$$

$$= 1 - \sin x = P \qquad \underline{L = P}$$

b) $\dfrac{\sin x}{1+\cot x} + \dfrac{\cos x}{1+\operatorname{tg} x} = \dfrac{1}{\sin x + \cos x}$

$$L = \frac{\sin x}{1+\cot x} + \frac{\cos x}{1+\operatorname{tg} x} = \frac{\sin x}{1+\frac{\cos x}{\sin x}} + \frac{\cos x}{1+\frac{\sin x}{\cos x}} =$$

$$= \frac{\sin x}{\frac{\sin x+\cos x}{\sin x}} + \frac{\cos x}{\frac{\cos x+\sin x}{\cos x}} = \frac{\sin x}{1}\cdot\frac{\sin x}{\sin x+\cos x} + \frac{\cos x}{1}\cdot\frac{\cos x}{\cos x+\sin x} =$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x} = \underline{\frac{1}{\sin x+\cos x}} = P \qquad \underline{L = P}$$

$\boxed{\text{Př 4}}$ Zjednodušte:

a) $\sin\left(\frac{\pi}{4}+x\right) - \sin\left(\frac{\pi}{4}-x\right) =$

$$= \sin\frac{\pi}{4}\cdot\cos x + \cos\frac{\pi}{4}\sin x - \left(\sin\frac{\pi}{4}\cdot\cos x - \cos\frac{\pi}{4}\sin x\right) =$$

$$= \cancel{\frac{\sqrt{2}}{2}\cdot\cos x} + \frac{\sqrt{2}}{2}\sin x - \cancel{\frac{\sqrt{2}}{2}\cos x} + \frac{\sqrt{2}}{2}\sin x = 2\frac{\sqrt{2}}{2}\sin x = \underline{\sqrt{2}\sin x}$$

b) $$\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\sin(\alpha+\beta)-\sin(\alpha-\beta)} = \frac{\sin\alpha\cdot\cos\beta + \cancel{\cos\alpha\sin\beta} + \sin\alpha\cos\beta - \cancel{\cos\alpha\sin\beta}}{\cancel{\sin\alpha\cos\beta} + \cos\alpha\sin\beta - \cancel{\sin\alpha\cos\beta} + \cos\alpha\sin\beta} =$$

$$= \frac{\cancel{2}\sin\alpha\cos\beta}{\cancel{2}\cos\alpha\sin\beta} = \underline{\operatorname{tg}\alpha\cdot\cot\beta} \qquad \sin\beta \neq 0 \quad \cos\alpha \neq 0$$

c) $\sin(\alpha+\beta)\cdot\cos\beta - \cos(\alpha+\beta)\cdot\sin\beta =$

$$= (\sin\alpha\cos\beta + \cos\alpha\sin\beta)\cdot\cos\beta - (\cos\alpha\cos\beta - \sin\alpha\sin\beta)\cdot\sin\beta =$$

$$= \sin\alpha\cdot\cos^2\beta + \cancel{\cos\alpha\sin\beta\cos\beta} - \cancel{\cos\alpha\cos\beta\sin\beta} + \sin\alpha\sin^2\beta =$$

$$= \sin\alpha\cdot\cos^2\beta + \sin\alpha\cdot\sin^2\beta = \sin\alpha(\underbrace{\cos^2\beta+\sin^2\beta}_{=1}) = \underline{\sin\alpha}$$

d) $\cos(20^\circ+x)\cdot\cos(20^\circ-x) + \sin(20^\circ+x)\sin(20^\circ-x) =$

$$= (\cos 20^\circ\cos x - \sin 20^\circ\sin x)(\cos 20^\circ\cos x + \sin 20^\circ\sin x) +$$

$$+ (\sin 20^\circ\cos x + \cos 20^\circ\sin x)\cdot(\sin 20^\circ\cos x - \cos 20^\circ\sin x) =$$

$$= (\cos 20^\circ\cos x)^2 - (\sin 20^\circ\sin x)^2 + (\sin 20^\circ\cos x)^2 - (\cos 20^\circ\sin x)^2 =$$

$$= \cos^2 x\cdot(\underbrace{\cos^2 20^\circ + \sin^2 20^\circ}_{1}) - \sin^2 x(\underbrace{\sin^2 20^\circ + \cos^2 20^\circ}_{1}) =$$

$$= \cos^2 x - \sin^2 x = \underline{\cos 2x}$$

Př 5 Určete bez použití MF tabulek nebo kalkulátoru:

a) $\sin 12^\circ \cdot \cos 18^\circ + \sin 18^\circ \cos 12^\circ = \sin(12^\circ + 18^\circ) = \sin 30^\circ = \frac{1}{2}$

$\sin\alpha \cdot \cos\beta + \sin\beta\cos\alpha$ $\qquad \sin(\alpha+\beta)$

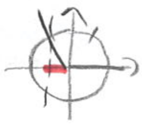
b) $\sin\frac{3}{7}\pi \cdot \sin\frac{5}{21}\pi - \cos\frac{3}{7}\pi \cdot \cos\frac{5}{21}\pi =$

$= (-1)\cdot\left(\cos\frac{3}{7}\pi\cos\frac{5}{21}\pi - \sin\frac{3}{7}\pi\cdot\sin\frac{5}{21}\pi\right) = (-1)\cdot\cos\left(\frac{3}{7}\pi + \frac{5}{21}\pi\right) =$

$\cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\qquad \cos(\alpha+\beta)$

$= (-1)\cdot\cos\frac{9\pi + 5\pi}{21} = (-1)\cdot\cos\frac{14}{21}\pi = (-1)\cdot\cos\frac{2}{3}\pi =$

$= (-1)\cdot\left(\frac{-1}{2}\right) = \frac{1}{2}$



c) $\dfrac{\sin(30^\circ+x) - \sin(30^\circ-x)}{\cos(60^\circ+x) + \cos(60^\circ-x)} =$

$= \dfrac{\cancel{\sin 30^\circ\cos x} + \cos 30^\circ\sin x - \cancel{\sin 30^\circ\cos x} + \cos 30^\circ\sin x}{\cos 60^\circ\cos x - \cancel{\sin 60^\circ\sin x} + \cos 60^\circ\cos x + \cancel{\sin 60^\circ\sin x}} =$

$= \dfrac{2\cos 30^\circ\sin x}{2\cos 60^\circ\cos x} = \dfrac{\cancel{2}\cdot\frac{\sqrt{3}}{\cancel{2}}\sin x}{\cancel{2}\cdot\frac{1}{\cancel{2}}\cos x} = \sqrt{3}\,\dfrac{\sin x}{\cos x} = \sqrt{3}\cdot\mathrm{tg}\,x$

Př 6 Zjednodušte (vypočtěte):

a) $\cos 15^\circ - \cos 75^\circ = -2\sin\frac{15^\circ+75^\circ}{2}\cdot\sin\frac{15^\circ-75^\circ}{2} = -2\cdot\sin 45^\circ\cdot\sin(-30^\circ) =$

$= -2\cdot\sin 45^\circ\cdot(-\sin 30^\circ) = -\cancel{2}\cdot\frac{\sqrt{2}}{\cancel{2}}\cdot\left(\frac{-1}{2}\right) = \frac{\sqrt{2}}{2}$

b) $(\sin 15^\circ + \sin 75^\circ)\cdot(\cos 75^\circ - \cos 15^\circ) =$

$= 2\sin\frac{15^\circ+75^\circ}{2}\cdot\cos\frac{15^\circ-75^\circ}{2}\cdot(-2)\cdot\sin\frac{75^\circ+15^\circ}{2}\cdot\sin\frac{75^\circ-15^\circ}{2} =$

$= 2\cdot\sin 45^\circ\cdot\cos(-30^\circ)\cdot(-2)\cdot\sin 45^\circ\cdot\sin 30^\circ =$

$= \cancel{2}\cdot\frac{\sqrt{2}}{\cancel{2}}\cdot\frac{\sqrt{3}}{2}\cdot(-\cancel{2})\cdot\frac{\sqrt{2}}{\cancel{2}}\cdot\frac{1}{2} = \frac{-2\sqrt{3}}{4} = \frac{-\sqrt{3}}{2}$

c) $\dfrac{\sin 5\alpha - \sin 3\alpha}{\cos 5\alpha + \cos 3\alpha} = \dfrac{\cancel{2\cos\frac{5\alpha+3\alpha}{2}}\cdot\sin\frac{5\alpha-3\alpha}{2}}{\cancel{2\cos\frac{5\alpha+3\alpha}{2}}\cdot\cos\frac{5\alpha-3\alpha}{2}} = \dfrac{\sin\frac{2\alpha}{2}}{\cos\frac{2\alpha}{2}} = \dfrac{\sin\alpha}{\cos\alpha} = \mathrm{tg}\,\alpha$

d) $\dfrac{\sin x + \sin 2x + \sin 3x}{\cos x + \cos 2x + \cos 3x} = \dfrac{2\sin\frac{x+3x}{2}\cdot\cos\frac{x-3x}{2} + \sin 2x}{2\cos\frac{x+3x}{2}\cdot\cos\frac{x-3x}{2} + \cos 2x} =$

$= \dfrac{2\sin 2x\cos(-x) + \sin 2x}{2\cos 2x\cos(-x) + \cos 2x} = \dfrac{\sin 2x\cdot\cancel{(2\cos x + 1)}}{\cos 2x\cdot\cancel{(2\cos x + 1)}} = \mathrm{tg}\,2x$

e) $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ = 2\cos\frac{50^\circ+70^\circ}{2}\cdot\sin\frac{50^\circ-70^\circ}{2} + \sin 10^\circ =$

$= 2\cos 60^\circ\cdot\sin(-10^\circ) + \sin 10^\circ = -\cancel{2}\cdot\frac{1}{\cancel{2}}\cdot\sin 10^\circ + \sin 10^\circ = 0$

f) $\cos 255^\circ = \cos(210^\circ + 45^\circ) = \cos 210^\circ\cdot\cos 45^\circ - \sin 210^\circ\sin 45^\circ =$

$= \frac{-\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{-1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{-\sqrt{6}+\sqrt{2}}{4}$

g) $\sin 345^\circ = \sin(300^\circ + 45^\circ) = \sin 300^\circ\cdot\cos 45^\circ + \sin 45^\circ\cdot\cos 300^\circ =$

$= \frac{-\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{-\sqrt{6}+\sqrt{2}}{4}$