GONIOMETRICKÉ ROVNICE – 2

Př1 Řešte následující rovnice:

a) $\sin^2 x + \cos x + 1 = 0$

$1 - \cos^2 x + \cos x + 1 = 0$

$-\cos^2 x + \cos x + 2 = 0$

<u>subst:</u> $y = \cos x$

$-y^2 + y + 2 = 0$

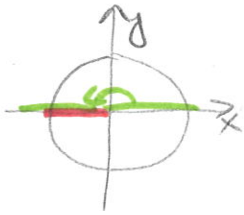
$D = 1 + 4 \cdot 2 = 1 + 8 = 9$

$y_{1,2} = \dfrac{-1 \pm \sqrt{9}}{-2} = \dfrac{-1 \pm 3}{-2}$

$y_1 = \dfrac{-1+3}{-2} = \dfrac{2}{-2} = (-1)$

$y_2 = \dfrac{-1-3}{-2} = \dfrac{-4}{-2} = 2$

$\cos x = (-1)$



$$\boxed{x = \pi + 2k\pi,\ k \in \mathbb{Z}}$$

$\cos x = 2$

$K' = \emptyset$

b) $\sin x + \cos^2 x = \dfrac{1}{4}$

$\sin x + 1 - \sin^2 x = \dfrac{1}{4} \quad / \cdot 4$

$4\sin x + 4 - 4\sin^2 x = 1$

$-4\sin^2 x + 4\sin x + 3 = 0$

<u>subst.</u> $y = \sin x$

$-4y^2 + 4y + 3 = 0$

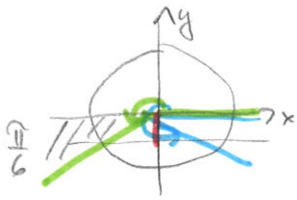
$D = 16 + 4 \cdot 4 \cdot 3 = 16 + 48 = 64$

$y_{1,2} = \dfrac{-4 \pm \sqrt{64}}{2 \cdot (-4)} = \dfrac{-4 \pm 8}{-8}$

$y_1 = \dfrac{-4+8}{-8} = \dfrac{4}{-8} = \dfrac{-1}{2}$

$y_2 = \dfrac{-4-8}{-8} = \dfrac{-12}{-8} = \dfrac{3}{2}$

$\sin x = \dfrac{-1}{2}$



$$\boxed{\begin{aligned} x_1 &= \tfrac{7}{6}\pi + 2k\pi,\ k \in \mathbb{Z} \\ x_2 &= \tfrac{11}{6}\pi + 2k\pi,\ k \in \mathbb{Z} \end{aligned}}$$

$\sin x = \dfrac{3}{2}$

$K' = \emptyset$

c) $3\cos^2 x - 4\cos x - \sin^2 x - 2 = 0$

$3\cos^2 x - 4\cos x - (1 - \cos^2 x) - 2 = 0$

$3\cos^2 x - 4\cos x - 1 + \cos^2 x - 2 = 0$

$4\cos^2 x - 4\cos x - 3 = 0$

subst: $y = \cos x$

$4y^2 - 4y - 3 = 0$

$D = 16 + 4\cdot 4\cdot 3 = 16 + 48 = 64$

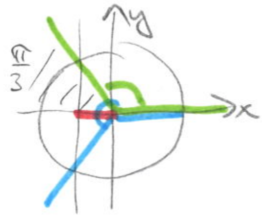
$y_{1,2} = \frac{4 \pm \sqrt{64}}{8} = \frac{4 \pm 8}{8}$

$y_1 = \frac{4+8}{8} = \frac{12}{8} = \frac{3}{2}$ $\quad y_2 = \frac{4-8}{8} = \frac{-4}{8} = \frac{-1}{2}$

$\cos x = \frac{3}{2}$

$K' = \emptyset$

$\cos x = \frac{-1}{2}$



$$\boxed{\begin{aligned} x_1 &= \tfrac{2}{3}\pi + 2k\pi, k \in \mathbb{Z} \\ x_2 &= \tfrac{4}{3}\pi + 2k\pi, k \in \mathbb{Z} \end{aligned}}$$

d) $\cos^2 x - 6\sin x + 6 = 0$

$1 - \sin^2 x - 6\sin x + 6 = 0$

$-\sin^2 x - 6\sin x + 7 = 0$

subst: $y = \sin x$

$-y^2 - 6y + 7 = 0$

$D = 36 + 28 = 64$

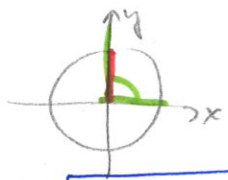
$y_{1,2} = \frac{6 \pm \sqrt{64}}{-2} = \frac{6 \pm 8}{-2}$

$y_1 = \frac{6+8}{-2} = \frac{14}{-2} = (-7)$ $\quad y_2 = \frac{6-8}{-2} = \frac{-2}{-2} = 1$

$\sin x = (-7)$

$K' = \emptyset$

$\sin x = 1$



$$\boxed{x = \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}}$$

e) $\sin^2 x - \cos^2 x = 0{,}5$

I. z. $1 - \cos^2 x - \cos^2 x = 0{,}5$

$-2\cos^2 x + 0{,}5 = 0$

subst: $y = \cos x$

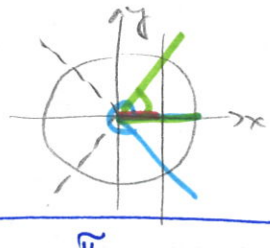
$-2y^2 + 0{,}5 = 0$

$D = 0 + 4 = 4$

$y_{1,2} = \frac{0 \pm \sqrt{4}}{-4} = \frac{\pm 2}{-4}$

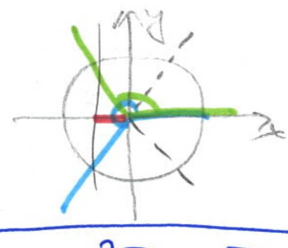
$y_1 = \frac{-2}{-4} = \frac{1}{2}$ $\quad y_2 = \frac{2}{-4} = \frac{-1}{2}$

$\cos x = \frac{1}{2}$



$$\boxed{\begin{aligned} x_1 &= \tfrac{\pi}{3} + 2k\pi, k \in \mathbb{Z} \\ x_2 &= \tfrac{5}{3}\pi + 2k\pi, k \in \mathbb{Z} \end{aligned}}$$

$\cos x = \frac{-1}{2}$

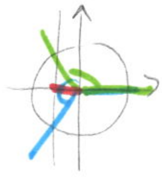


$$\boxed{\begin{aligned} x_3 &= \tfrac{2}{3}\pi + 2k\pi, k \in \mathbb{Z} \\ x_4 &= \tfrac{4}{3}\pi + 2k\pi, k \in \mathbb{Z} \end{aligned}}$$

II. z. $(-1)\cdot(\cos^2 x - \sin^2 x) = 0{,}5$

$(-1)\cdot \cos 2x = 0{,}5$

$\cos 2x = \frac{-1}{2}$



$2x = \frac{2}{3}\pi + 2k\pi \;/:2$

$$\boxed{x_1 = \frac{\pi}{3} + k\pi, k \in \mathbb{Z}}$$

$2x = \frac{4}{3}\pi + 2k\pi \;/:2$

$$\boxed{x_2 = \frac{2}{3}\pi + k\pi, k \in \mathbb{Z}}$$

Př 2 Řešte následující rovnice:

a) $\sin 5x = \sin 3x$

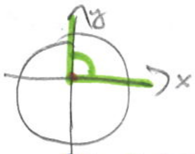
$$\sin 5x - \sin 3x = 0$$

$$2\cos\frac{5x+3x}{2}\cdot\sin\frac{5x-3x}{2} = 0$$

$$2\cos 4x\cdot\sin x = 0$$

$\swarrow$ $\longrightarrow$

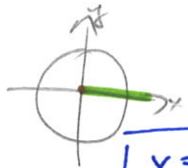
$\cos 4x = 0$



$$4x = \frac{\pi}{2} + k\pi \quad /:4$$

$$\boxed{x_1 = \frac{\pi}{8} + \frac{k\pi}{4},\ k\in\mathbb{Z}}$$

$\sin x = 0$



$$\boxed{x_2 = k\pi,\ k\in\mathbb{Z}}$$

b) $\sin x = \sin 2x$

$$\sin x - \sin 2x = 0$$

$$\sin x - 2\sin x\cos x = 0$$

$$\sin x\cdot(1-2\cos x) = 0$$

$\swarrow$ $\longrightarrow$

$\sin x = 0$

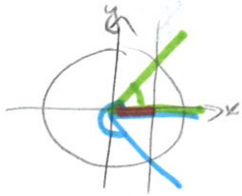
$$\boxed{x_1 = k\pi,\ k\in\mathbb{Z}}$$

$$1-2\cos x = 0$$

$$-2\cos x = -1$$

$$2\cos x = 1$$

$$\cos x = \frac{1}{2}$$



$$\boxed{\begin{aligned} x_2 &= \frac{\pi}{3} + 2k\pi,\ k\in\mathbb{Z} \\ x_3 &= \frac{5}{3}\pi + 2k\pi,\ k\in\mathbb{Z} \end{aligned}}$$

c) $\cos\left(x+\frac{\pi}{6}\right) - \cos\left(x-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

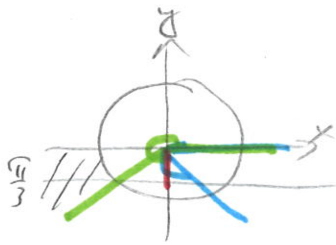
$$\cancel{\cos x\cdot\cos\frac{\pi}{6}} - \sin x\cdot\sin\frac{\pi}{6} - \cancel{\cos x\cdot\cos\frac{\pi}{6}} - \sin x\cdot\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$-2\sin x\cdot\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$-\cancel{2}\sin x\cdot\frac{1}{\cancel{2}} = \frac{\sqrt{3}}{2}$$

$$-\sin x = \frac{\sqrt{3}}{2}$$

$$\sin x = \frac{-\sqrt{3}}{2}$$



$$\boxed{\begin{aligned} x_1 &= \frac{4}{3}\pi + 2k\pi,\ k\in\mathbb{Z} \\ x_2 &= \frac{5}{3}\pi + 2k\pi,\ k\in\mathbb{Z} \end{aligned}}$$

d) $\sin\left(x+\frac{\pi}{4}\right) - \sin\left(x-\frac{\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

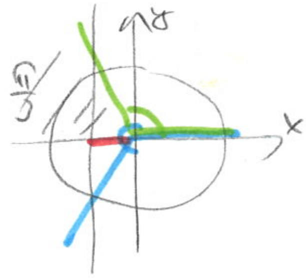
$$\cancel{\sin x \cdot \cos\frac{\pi}{4}} + \cos x \cdot \sin\frac{\pi}{4} - \cancel{\sin x \cdot \cos\frac{\pi}{4}} + \cos x \cdot \sin\frac{\pi}{4} = \frac{-\sqrt{2}}{2}$$

$$2\cos x \cdot \sin\frac{\pi}{4} = \frac{-\sqrt{2}}{2}$$

$$\cancel{2}\cos x \cdot \frac{\sqrt{2}}{\cancel{2}} = \frac{-\sqrt{2}}{2}$$

$$\sqrt{2}\cdot\cos x = \frac{-\sqrt{2}}{2} \quad /:\sqrt{2}$$

$$\cos x = \frac{-1}{2}$$



$$\boxed{\begin{aligned} x_1 &= \tfrac{2}{3}\pi + 2k\pi, \; k\in\mathbb{Z} \\ x_2 &= \tfrac{4}{3}\pi + 2k\pi, \; k\in\mathbb{Z}\end{aligned}}$$

e) $\sin\alpha + \cos 2\alpha = 1$

$$\sin\alpha + \cos^2\alpha - \sin^2\alpha = 1$$

$$\sin\alpha + \cancel{1} - \sin^2\alpha - \sin^2\alpha = \cancel{1}$$

$$-2\sin^2\alpha + \sin\alpha = 0$$

$$\sin\alpha\cdot(-2\sin\alpha + 1) = 0 \longrightarrow -2\sin\alpha + 1 = 0$$

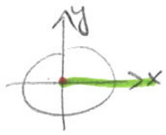
$$-2\sin\alpha = -1$$

$$\sin\alpha = \frac{1}{2}$$

$$\boxed{\begin{aligned}\alpha_2 &= \frac{\pi}{6} + 2k\pi, \; k\in\mathbb{Z} \\ \alpha_3 &= \frac{5}{6}\pi + 2k\pi, \; k\in\mathbb{Z}\end{aligned}}$$

$\swarrow$

$$\sin\alpha = 0$$



$$\boxed{\alpha_1 = k\pi, \; k\in\mathbb{Z}}$$

f) $\sin 3x - \sin 2x = 0$

$$2\cos\frac{3x+2x}{2}\cdot\sin\frac{3x-2x}{2} = 0$$

$$2\cos\frac{5x}{2}\cdot\sin\frac{x}{2} = 0$$

$\swarrow$ $\longrightarrow$ $\sin\frac{x}{2} = 0$

$$\cos\frac{5x}{2} = 0$$

$$\frac{x}{2} = k\pi$$

$$\boxed{x_2 = 2k\pi, \; k\in\mathbb{Z}}$$

$$\frac{5x}{2} = \frac{\pi}{2} + k\pi$$

$$5x = \pi + 2k\pi$$

$$\boxed{x_1 = \frac{\pi}{5} + \frac{2k\pi}{5}, \; k\in\mathbb{Z}}$$