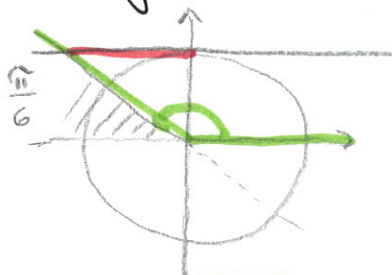


GONIOMETRICKÉ ROVNICE - ①

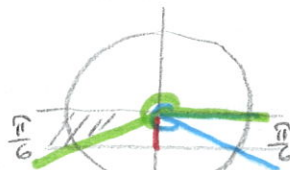
Pr 1 Řešte následující goniometrické rovnice:

a) $\cot x = -\sqrt{3}$



$$x = \frac{5}{6}\pi + k\pi, k \in \mathbb{Z}$$

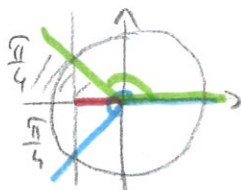
b) $\sin x = -\frac{1}{2}$



$$x_1 = \frac{7}{6}\pi + 2k\pi$$

$$x_2 = \frac{11}{6}\pi + 2k\pi, k \in \mathbb{Z}$$

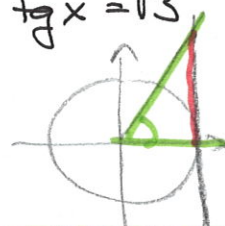
c) $\cos x = -\frac{\sqrt{2}}{2}$



$$x_1 = \frac{3}{4}\pi + 2k\pi$$

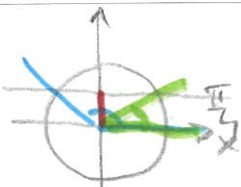
$$x_2 = \frac{5}{4}\pi + 2k\pi, k \in \mathbb{Z}$$

d) $\tan x = \sqrt{3}$



$$x = \frac{\pi}{3} + k\pi, k \in \mathbb{Z}$$

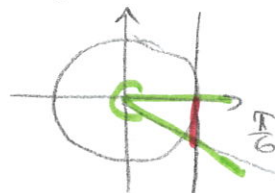
e) $\sin x = \frac{\sqrt{2}}{2}$



$$x_1 = \frac{\pi}{4} + 2k\pi$$

$$x_2 = \frac{3}{4}\pi + 2k\pi, k \in \mathbb{Z}$$

f) $\tan x = -\frac{\sqrt{3}}{3}$

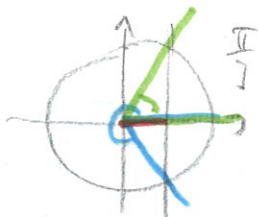


$$x = \frac{11}{6}\pi + k\pi, k \in \mathbb{Z}$$

nebo

$$x = \frac{5}{6}\pi + k\pi, k \in \mathbb{Z}$$

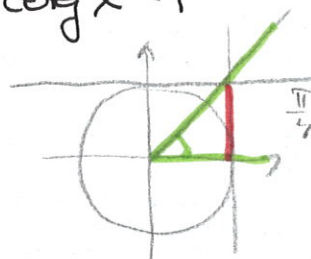
g) $\cos x = \frac{1}{2}$



$$x_1 = \frac{\pi}{3} + 2k\pi$$

$$x_2 = \frac{5}{3}\pi + 2k\pi, k \in \mathbb{Z}$$

h) $\cot x = 1$

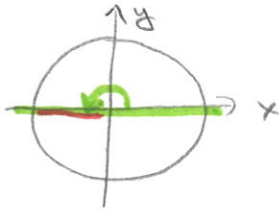


$$x = \frac{\pi}{4} + k\pi, k \in \mathbb{Z}$$

$$e) \cos(2x - \frac{\pi}{4}) = -1$$

$$\text{subst: } y = 2x - \frac{\pi}{4}$$

$$\cos y = -1$$



$$y = \pi + 2k\pi$$

$$2x - \frac{\pi}{4} = \pi + 2k\pi$$

$$2x = \pi + \frac{\pi}{4} + 2k\pi$$

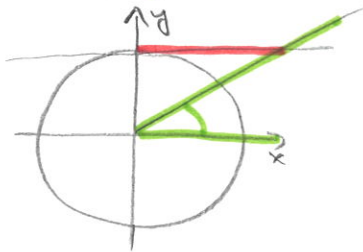
$$2x = \frac{5\pi}{4} + 2k\pi$$

$$x = \frac{5\pi}{8} + k\pi, k \in \mathbb{Z}$$

$$f) \cotg(\frac{x}{2} - \frac{\pi}{3}) = \sqrt{3}$$

$$\text{subst: } y = \frac{x}{2} - \frac{\pi}{3}$$

$$\cotg y = \sqrt{3}$$



$$y = \frac{\pi}{6} + k\pi$$

$$\frac{x}{2} - \frac{\pi}{3} = \frac{\pi}{6} + k\pi$$

$$\frac{x}{2} = \frac{\pi}{6} + \frac{\pi}{3} + k\pi$$

$$\frac{x}{2} = \frac{\pi + 2\pi}{6} + k\pi$$

$$\frac{x}{2} = \frac{3\pi}{6} + k\pi \quad / \cdot 2$$

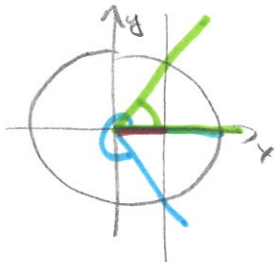
$$x = \frac{6\pi}{6} + 2k\pi$$

$$x = \pi + 2k\pi, k \in \mathbb{Z}$$

$$g) \cos(3x + \frac{\pi}{2}) = \frac{\sqrt{2}}{2}$$

$$\text{subst: } y = 3x + \frac{\pi}{2}$$

$$\cos y = \frac{\sqrt{2}}{2}$$



$$y_1 = \frac{\pi}{4} + 2k\pi$$

$$3x + \frac{\pi}{2} = \frac{\pi}{4} + 2k\pi$$

$$3x = \frac{\pi}{4} - \frac{\pi}{2} + 2k\pi$$

$$3x = \frac{\pi - 2\pi}{4} + 2k\pi$$

$$3x = \frac{-\pi}{4} + 2k\pi \quad / : 3$$

$$x_1 = \frac{-\pi}{12} + \frac{2k\pi}{3}, k \in \mathbb{Z}$$

$$\text{webo} \quad x_1 = \frac{23\pi}{12} + \frac{2k\pi}{3}, k \in \mathbb{Z}$$

$$y_2 = \frac{7\pi}{4} + 2k\pi$$

$$3x + \frac{\pi}{2} = \frac{7\pi}{4} + 2k\pi$$

$$3x = \frac{7\pi}{4} - \frac{\pi}{2} + 2k\pi$$

$$3x = \frac{7\pi - 2\pi}{4} + 2k\pi$$

$$3x = \frac{5\pi}{4} + 2k\pi \quad / : 3$$

$$x_2 = \frac{5\pi}{12} + \frac{2k\pi}{3}, k \in \mathbb{Z}$$

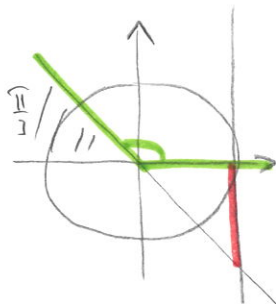
$$h) \frac{2}{\sqrt{3}} \cdot \tg(3x - \frac{\pi}{4}) = -2$$

$$\text{subst: } y = 3x - \frac{\pi}{4}$$

$$\frac{2}{\sqrt{3}} \cdot \tg y = -2 \quad / \cdot \frac{\sqrt{3}}{2}$$

$$\tg y = \frac{-2\sqrt{3}}{2}$$

$$\tg y = -\sqrt{3}$$



$$y = \frac{2\pi}{3} + k\pi$$

$$3x - \frac{\pi}{4} = \frac{2\pi}{3} + k\pi$$

$$3x = \frac{2\pi}{3} + \frac{\pi}{4} + k\pi$$

$$3x = \frac{8\pi + 3\pi}{12} + k\pi$$

$$3x = \frac{11\pi}{12} + k\pi \quad / : 3$$

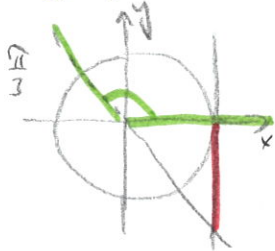
$$x = \frac{11\pi}{36} + \frac{k\pi}{3}, k \in \mathbb{Z}$$

Pr2 Řešte tyto goniometrické rovnice:

a) $\operatorname{tg}\left(\frac{\pi}{6} - x\right) = -\sqrt{3}$

subst. $y = \frac{\pi}{6} - x$

$\operatorname{tg} y = -\sqrt{3}$



$y = \frac{2}{3}\pi + k\pi$

$\frac{\pi}{6} - x = \frac{2}{3}\pi + k\pi$

$-x = \frac{2}{3}\pi - \frac{\pi}{6} + k\pi$

$-x = \frac{4\pi - \pi}{6} + k\pi$

$-x = \frac{3\pi}{6} + k\pi$

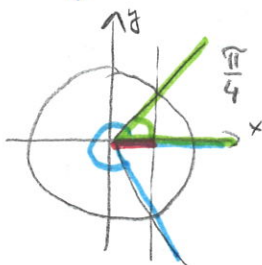
$-x = \frac{\pi}{2} + k\pi$

$x = -\frac{\pi}{2} - k\pi, k \in \mathbb{Z}$ nebo $x = \frac{3}{2}\pi + k\pi, k \in \mathbb{Z}$

b) $\cos\left(\frac{\pi}{4} + x\right) = \frac{\sqrt{2}}{2}$

subst. $y = \frac{\pi}{4} + x$

$\cos y = \frac{\sqrt{2}}{2}$



$y_1 = \frac{\pi}{4} + 2k\pi$

$\frac{\pi}{4} + x = \frac{\pi}{4} + 2k\pi$

$x_1 = 2k\pi, k \in \mathbb{Z}$

$y_2 = \frac{7}{4}\pi + 2k\pi$

$\frac{\pi}{4} + x = \frac{7}{4}\pi + 2k\pi$

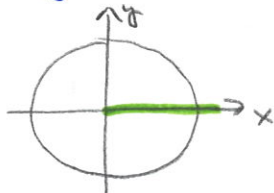
$x = \frac{6}{4}\pi + 2k\pi$

$x_2 = \frac{3}{2}\pi + 2k\pi, k \in \mathbb{Z}$

c) $\sin\left(x + \frac{\pi}{3}\right) = 0$

subst. $y = x + \frac{\pi}{3}$

$\sin y = 0$



$y = k\pi$

$x + \frac{\pi}{3} = k\pi$

$x = -\frac{\pi}{3} + k\pi, k \in \mathbb{Z}$

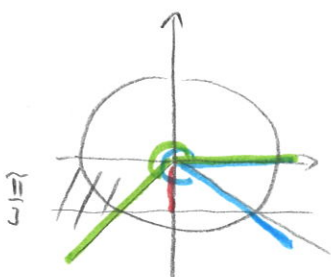
nebo $k \in \mathbb{Z}$

$x = \frac{5}{3}\pi + k\pi, k \in \mathbb{Z}$

d) $\sin\left(2x + \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

subst. $y = 2x + \frac{\pi}{3}$

$\sin y = -\frac{\sqrt{3}}{2}$



$y_1 = \frac{4}{3}\pi + 2k\pi$

$2x + \frac{\pi}{3} = \frac{4}{3}\pi + 2k\pi$

$2x = \frac{3}{3}\pi + 2k\pi$

$2x = \pi + 2k\pi$

$x_1 = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

$y_2 = \frac{5}{3}\pi + 2k\pi$

$2x + \frac{\pi}{3} = \frac{5}{3}\pi + 2k\pi$

$2x = \frac{4}{3}\pi + 2k\pi$

$x = \frac{2}{3}\pi + k\pi$

$x_2 = \frac{2}{3}\pi + k\pi, k \in \mathbb{Z}$

Pr3 Řešte následující rovnice:

a) $\cos^2 x + \cos x = 0$

subst: $y = \cos x$

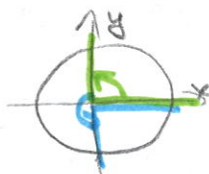
$y^2 + y = 0$

$y(y+1) = 0$

$\swarrow$
 $y_1 = 0$

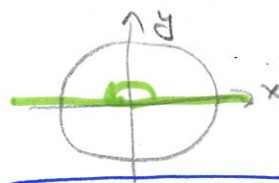
$\searrow$ $y+1=0$
 $y_2 = -1$

$\cos x = 0$



$x_1 = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

$\cos x = (-1)$



$x_2 = \pi + 2k\pi, k \in \mathbb{Z}$

b) $2 \sin^2 x = \sqrt{2} \sin x$

$2 \sin^2 x - \sqrt{2} \sin x = 0$

subst: $y = \sin x$

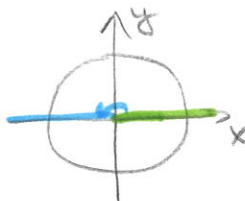
$2y^2 - \sqrt{2}y = 0$

$y(2y - \sqrt{2}) = 0$

$\swarrow$ $y_1 = 0$

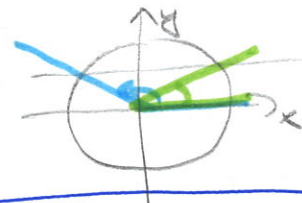
$\searrow$ $2y - \sqrt{2} = 0$
 $2y = \sqrt{2}$
 $y_2 = \frac{\sqrt{2}}{2}$

$\sin x = 0$



$x_1 = k\pi, k \in \mathbb{Z}$

$\sin x = \frac{\sqrt{2}}{2}$



$x_2 = \frac{\pi}{4} + 2k\pi, k \in \mathbb{Z}$

$x_3 = \frac{3}{4}\pi + 2k\pi, k \in \mathbb{Z}$

c) $3 \tan^2 x - \sqrt{3} \tan x = 0$

subst: $y = \tan x$

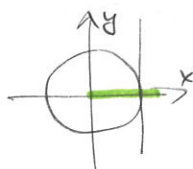
$3y^2 - \sqrt{3}y = 0$

$y(3y - \sqrt{3}) = 0$

$\swarrow$ $y_1 = 0$

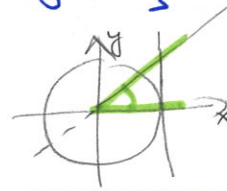
$\searrow$ $3y - \sqrt{3} = 0$
 $3y = \sqrt{3}$
 $y_2 = \frac{\sqrt{3}}{3}$

$\tan x = 0$



$x_1 = k\pi, k \in \mathbb{Z}$

$\tan x = \frac{\sqrt{3}}{3}$



$x_2 = \frac{\pi}{6} + k\pi, k \in \mathbb{Z}$

d) $\sqrt{3} \cot^2 x + 3 \cot x = 0$

subst: $y = \cot x$

$\sqrt{3}y^2 + 3y = 0$

$y(\sqrt{3}y + 3) = 0$

$\swarrow$ $y_1 = 0$

$\searrow$ $\sqrt{3}y + 3 = 0$

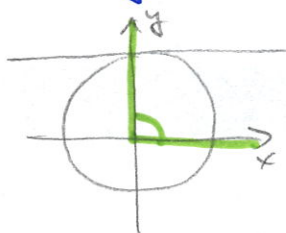
$\sqrt{3}y = -3$

$y = \frac{-3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$y = \frac{-3\sqrt{3}}{3}$

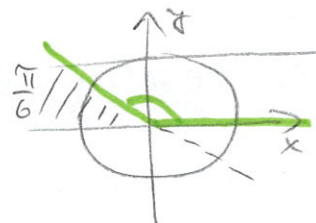
$y_2 = -\sqrt{3}$

$\cot x = 0$



$x_1 = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

$\cot x = -\sqrt{3}$



$x_2 = \frac{5}{6}\pi + k\pi, k \in \mathbb{Z}$

Pr4 Řešte tyto goniometrické rovnice:

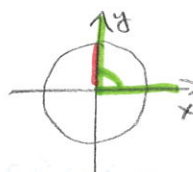
a) $2\sin^2 x - 3\sin x + 1 = 0$

subst: $y = \sin x$
 $2y^2 - 3y + 1 = 0$

$D = 9 - 4 \cdot 2 = 9 - 8 = 1$

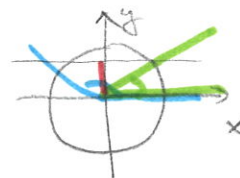
$y_{1,2} = \frac{3 \pm \sqrt{1}}{2 \cdot 2} = \frac{3 \pm 1}{4} \rightarrow y_1 = \frac{3+1}{4} = \frac{4}{4} = 1$
 $y_2 = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$

$\sin x = 1$



$x_1 = \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$

$\sin x = \frac{1}{2}$



$x_2 = \frac{\pi}{6} + 2k\pi, k \in \mathbb{Z}$

$x_3 = \frac{5}{6}\pi + 2k\pi, k \in \mathbb{Z}$

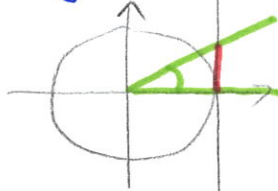
b) $\tan^2 x - 2\tan x + 1 = 0$

subst: $y = \tan x$
 $y^2 - 2y + 1 = 0$

$D = 4 - 4 \cdot 1 = 4 - 4 = 0$

$y = \frac{2 \pm \sqrt{0}}{2} = \frac{2}{2} = 1$

$\tan x = 1$



$x = \frac{\pi}{4} + k\pi, k \in \mathbb{Z}$

c) $2\cos^2 x + 5\cos x - 3 = 0$

subst: $y = \cos x$
 $2y^2 + 5y - 3 = 0$

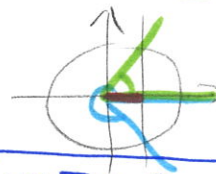
$D = 25 + 4 \cdot 2 \cdot 3 = 25 + 24 = 49$

$y = \frac{-5 \pm \sqrt{49}}{2 \cdot 2} = \frac{-5 \pm 7}{4} \rightarrow y_1 = \frac{-5+7}{4} = \frac{2}{4} = \frac{1}{2}$
 $y_2 = \frac{-5-7}{4} = \frac{-12}{4} = -3$

$\cos x = (-3)$

$x' = \emptyset$

$\cos x = \frac{1}{2}$



$x_1 = \frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$

$x_2 = \frac{5}{3}\pi + 2k\pi, k \in \mathbb{Z}$

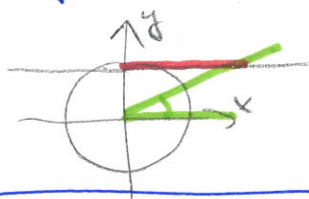
d) $\sqrt{3}\cot^2 x - 4\cot x + \sqrt{3} = 0$

subst: $y = \cot x$
 $\sqrt{3}y^2 - 4y + \sqrt{3} = 0$

$D = 16 - 4 \cdot \sqrt{3} \cdot \sqrt{3} = 16 - 4 \cdot 3 = 16 - 12 = 4$

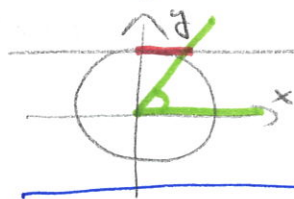
$y_{1,2} = \frac{4 \pm \sqrt{4}}{2\sqrt{3}} = \frac{4 \pm 2}{2\sqrt{3}} \rightarrow y_1 = \frac{4+2}{2\sqrt{3}} = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$
 $y_2 = \frac{4-2}{2\sqrt{3}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\cot x = \sqrt{3}$



$x_1 = \frac{\pi}{6} + k\pi, k \in \mathbb{Z}$

$\cot x = \frac{\sqrt{3}}{3}$



$x_2 = \frac{\pi}{3} + k\pi, k \in \mathbb{Z}$

e) $\sin^2 x + 5 \sin x + 4 = 0$

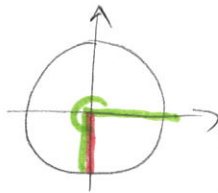
subst: $y = \sin x$

$y^2 + 5y + 4 = 0$

$D = 25 - 4 \cdot 4 = 25 - 16 = 9$

$y_{1,2} = \frac{-5 \pm \sqrt{9}}{2} = \frac{-5 \pm 3}{2} \rightarrow y_1 = \frac{-5+3}{2} = \frac{-2}{2} = \underline{\underline{-1}}$
 $y_2 = \frac{-5-3}{2} = \frac{-8}{2} = \underline{\underline{-4}}$

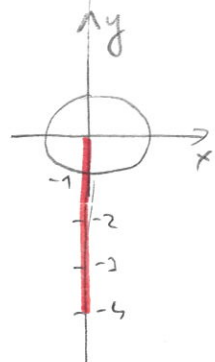
$\sin x = (-1)$



$x = \frac{3}{2}\pi + 2k\pi, k \in \mathbb{Z}$

$\sin x = (-4)$

$K' = \emptyset$



f) $\sin^2 x + 2 \sin x = 3$

subst: $y = \sin x$

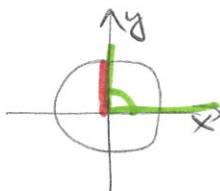
$y^2 + 2y = 3$

$y^2 + 2y - 3 = 0$

$D = 4 + 4 \cdot 3 = 4 + 12 = 16$

$y_{1,2} = \frac{-2 \pm \sqrt{16}}{2} = \frac{-2 \pm 4}{2} \rightarrow y_1 = \frac{-2+4}{2} = \frac{2}{2} = \underline{\underline{1}}$
 $y_2 = \frac{-2-4}{2} = \frac{-6}{2} = \underline{\underline{-3}}$

$\sin x = 1$



$x = \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$

$\sin x = (-3)$

$K' = \emptyset$