

LINEÁRNÍ ROVNICE (část 2.)

Př5 Řešte v $\mathbb{R}$ rovnice:

$$a) \frac{x+3}{x+5} = 3 - \frac{1-2x}{3-x} \quad / \cdot (x+5)(3-x) \quad \begin{matrix} x \neq -5 \\ x \neq 3 \end{matrix}$$

$$(x+3)(3-x) = 3(x+5)(3-x) - (1-2x)(x+5)$$

$$3x - x^2 + 9 - 3x = 3(3x - x^2 + 15 - 5x) - (x + 5 - 2x^2 - 10x)$$

$$-x^2 + 9 = 9x - 3x^2 + 45 - 15x - x - 5 + 2x^2 + 10x$$

$$-x^2 - 9x + 3x^2 + 15x + x - 2x^2 - 10x = 45 - 5 - 9$$

$$-3x = 31 \quad / : (-3)$$

$$x = \frac{-31}{3}$$

$$K = \left\{ \frac{-31}{3} \right\}$$

$$b) \frac{6}{x+2} + \frac{x+2}{2-x} + \frac{x^2}{x^2-4} = 0 \quad / \cdot (x+2)(x-2) \quad x \neq \pm 2$$
$$\frac{6}{(-1)(-2+x)} + \frac{(x+2)}{(x+2)(x-2)} + \frac{x^2}{(x+2)(x-2)} = 0$$

$$6(x-2) + (-1)(x+2)(x+2) + x^2 = 0$$

$$6x - 12 - (x^2 + 4x + 4) + x^2 = 0$$

$$6x - 12 - x^2 - 4x - 4 + x^2 = 0$$

$$6x - 4x = 12 + 4$$

$$2x = 16 \quad / : 2$$

$$x = 8$$

$$K = \{8\}$$

$$c) \frac{4}{x+2} + \frac{7}{x+3} = \frac{4}{x^2+5x+6} \quad / \cdot (x+2)(x+3) \quad \begin{matrix} x \neq -2 \\ x \neq -3 \end{matrix}$$
$$\frac{4}{(x+2)(x+3)} + \frac{7}{(x+2)(x+3)} = \frac{4}{(x+2)(x+3)}$$

$$4(x+3) + 7(x+2) = 4$$

$$4x + 12 + 7x + 14 = 4$$

$$4x + 7x = 4 - 12 - 14$$

$$11x = -22 \quad / : 11$$

$$x = (-2)$$

$\rightarrow$ ale podmínka $x \neq (-2)$!

$$K = \emptyset$$

$$d) \frac{2x-5}{3x-4} - \frac{4x-5}{6x-1} = 0 \quad / \cdot (3x-4)(6x-1) \quad \begin{matrix} x \neq \frac{4}{3} \\ x \neq \frac{1}{6} \end{matrix}$$

$$(2x-5)(6x-1) - (4x-5)(3x-4) = 0$$

$$12x^2 - 2x - 30x + 5 - (12x^2 - 16x - 15x + 20) = 0$$

$$12x^2 - 32x + 5 - 12x^2 + 16x + 15x - 20 = 0$$

$$-32x + 16x + 15x = 20 - 5$$

$$-x = 15$$

$$x = (-15)$$

$$K = \{-15\}$$

Pr 6Řešte v $\mathbb{R}$ rovnice:

$$a) \frac{11+3x}{x+3} - \frac{5x}{x-4} + \frac{x}{x^2-x-12} + 2 = 0 \quad / \cdot (x+3)(x-4)$$

$$x \neq -3$$

$$x \neq 4$$

$$(11+3x)(x-4) - 5x(x+3) + x + 2(x+3)(x-4) = 0$$

$$11x - 44 + 3x^2 - 12x - 5x^2 - 15x + x + 2(x^2 - x - 12) = 0$$

$$11x - 44 + 3x^2 - 12x - 5x^2 - 15x + x + 2x^2 - 2x - 24 = 0$$

$$11x - 12x - 15x + x - 2x = 24 + 44$$

$$-17x = 68 \quad / : (-17)$$

$$\underline{\underline{x = -4}}$$

$$\boxed{K = \{-4\}}$$

$$b) \frac{2+x}{x-1} - \frac{8}{3x-3} = \frac{5}{2x-2} + \frac{5}{18} \quad / \cdot 18 \cdot (x-1) \quad x \neq 1$$

$$3(x-1) \quad 2(x-1)$$

$$18(2+x) - 6 \cdot 8 = 9 \cdot 5 + 5 \cdot (x-1)$$

$$36 + 18x - 48 = 45 + 5x - 5$$

$$18x - 5x = 45 - 5 + 48 - 36$$

$$13x = 52 \quad / : 13$$

$$\underline{\underline{x = 4}}$$

$$\boxed{K = \{4\}}$$

$$c) \frac{1}{x-2} + \frac{x-5}{3x-6} = \frac{2}{3} \quad / \cdot 3(x-2) \quad x \neq 2$$

$$3(x-2)$$

$$3 + x - 5 = 2(x-2)$$

$$3 + x - 5 = 2x - 4$$

$$x - 2x = -4 + 5 - 3$$

$$-x = -2 \quad / \cdot (-1)$$

$$\underline{\underline{x = 2}}$$

ale podm. $x \neq 2$

$$\boxed{K = \emptyset}$$

$$d) \frac{3+4x}{x^2+x} - 1 = \frac{3}{x} - \frac{x}{x+1} \quad / \cdot x \cdot (x+1) \quad x \neq 0$$

$$x(x+1) \quad x \neq -1$$

$$3+4x - (x^2+x) = 3(x+1) - x^2$$

$$3+4x - x^2 - x = 3x+3 - x^2$$

$$4x - x^2 - x - 3x + x^2 = 3 - 3$$

$$\underline{\underline{0 = 0}}$$

$$\boxed{K = \mathbb{R} - \{-1; 0\}}$$

Př 7 Řešte v $\mathbb{R}$ rovnice:

$$a) \frac{2x}{x+3} - \frac{2x}{x-3} = \frac{72}{4x^2-36} \quad / \cdot 4(x+3)(x-3) \quad x \neq \pm 3$$

$$4(x^2-9) = 4(x+3)(x-3)$$

$$8x(x-3) - 8x(x+3) = 72$$

$$8x^2 - 24x - 8x^2 - 24x = 72$$

$$-48x = 72 \quad / : (-48)$$

$$x = \frac{-72}{48} = -\frac{3}{2}$$

$$x = -\frac{3}{2}$$

$$K = \left\{ -\frac{3}{2} \right\}$$

$$b) \frac{1}{x-2} + x = \frac{(x-1)^2}{x-2} \quad / \cdot (x-2) \quad x \neq 2$$

$$1 + x(x-2) = (x-1)^2$$

$$1 + x^2 - 2x = x^2 - 2x + 1$$

$$x^2 - 2x - x^2 + 2x = 1 - 1$$

$$0 = 0$$

$$K = \mathbb{R} - \{2\}$$

$$c) \frac{8x}{2x+3} + \frac{3}{x} = \frac{3}{2x^2+3x} + 4 \quad / \cdot x \cdot (2x+3) \quad x \neq 0, x \neq -\frac{3}{2}$$

$$x(2x+3)$$

$$8x^2 + 3(2x+3) = 3 + 4x(2x+3)$$

$$8x^2 + 6x + 9 = 3 + 8x^2 + 12x$$

$$8x^2 + 6x - 8x^2 - 12x = 3 - 9$$

$$-6x = -6 \quad / : (-6)$$

$$x = 1$$

$$K = \{1\}$$

$$d) \frac{3x+1}{5x-2} = \frac{2(3x+14)}{5(2x+7)} \quad / \cdot 5 \cdot (5x-2)(2x+7) \quad x \neq -\frac{7}{2}, x \neq \frac{2}{5}$$

$$5(3x+1)(2x+7) = 2(3x+14)(5x-2)$$

$$5(6x^2 + 21x + 2x + 7) = 2(15x^2 - 6x + 70x - 28)$$

$$30x^2 + 105x + 10x + 35 = 30x^2 - 12x + 140x - 56$$

$$30x^2 + 105x + 10x - 30x^2 + 12x - 140x = -56 - 35$$

$$-13x = -91 \quad / : (-13)$$

$$x = 7$$

$$K = \{7\}$$

Př 8 Řešte v $\mathbb{R}$ rovnice:

$$a) \frac{x-2}{x+2} - \frac{x+2}{x-2} + \frac{8x}{x^2-4} = 0 \quad / \cdot (x-2)(x+2) \quad x \neq \pm 2$$

$$(x-2)(x-2) - (x+2)(x+2) + 8x = 0$$

$$x^2 - 4x + 4 - (x^2 + 4x + 4) + 8x = 0$$

$$x^2 - 4x + 4 - x^2 - 4x - 4 + 8x = 0$$

$$-4x - 4x + 8x = 0$$

$$0 = 0$$

$$K = \mathbb{R} - \{-2, 2\}$$

$$b) \frac{3}{1-x^2} = \frac{2}{(1+x)^2} - \frac{5}{(1-x)^2} \quad | \cdot (1+x)^2(1-x)^2 \quad x \neq \pm 1$$

$$3(1+x)(1-x) = 2(1-x)^2 - 5(1+x)^2$$

$$3(1-x^2) = 2(1-2x+x^2) - 5(1+2x+x^2)$$

$$3-3x^2 = 2-4x+2x^2-5-10x-5x^2$$

$$-3x^2+4x-2x^2+10x+5x^2 = 2-5-3$$

$$14x = -6$$

$$x = \underline{\underline{-\frac{3}{7}}}$$

$\div: 14$

$$K = \left\{ -\frac{3}{7} \right\}$$

Pr 9 Řešte v $\mathbb{R}$ rovnice:

$$a) \frac{\frac{1}{2}x - \frac{2}{3}}{\frac{3}{4}x + \frac{4}{3}} = \frac{\frac{5}{6}x - \frac{1}{3}}{\frac{5}{4}x + \frac{1}{5}}$$

$$\frac{\frac{3x-4}{6}}{\frac{9x+16}{12}} = \frac{\frac{5x-2}{6}}{\frac{25x+4}{20}}$$

$$\frac{3x-4}{6} : \frac{9x+16}{12} = \frac{5x-2}{6} : \frac{25x+4}{20}$$

$$\frac{3x-4}{6} \cdot \frac{12}{9x+16} = \frac{5x-2}{6} \cdot \frac{20}{25x+4} \quad | \cdot 3(9x+16)(25x+4)$$

$$x \neq -\frac{16}{9}$$

$$x \neq -\frac{4}{25}$$

$$3(3x-4) \cdot 2 \cdot (25x+4) = (5x-2) \cdot 10(9x+16)$$

$$6(3x-4)(25x+4) = 10(5x-2)(9x+16)$$

$$6 \cdot (75x^2 + 12x - 100x - 16) = 10 \cdot (45x^2 + 80x - 18x - 32)$$

$$450x^2 + 72x - 600x - 96 = 450x^2 + 800x - 180x - 320$$

$$72x - 600x - 800x + 180x = -320 + 96$$

$$-1148x = -224 \quad | : (-1148)$$

$$x = \frac{224}{1148}$$

$$x = \frac{56}{287} = \underline{\underline{\frac{8}{41}}}$$

$$K = \left\{ \frac{8}{41} \right\}$$

$$b) \frac{\frac{x}{2}-2}{x-1} + \frac{\frac{x}{2}+2}{x+1} = 1 \quad | \cdot (x-1)(x+1) \quad x \neq \pm 1$$

$$\left(\frac{x}{2}-2\right)(x+1) + \left(\frac{x}{2}+2\right)(x-1) = (x-1)(x+1)$$

$$\frac{x-4}{2} \cdot (x+1) + \frac{x+4}{2} \cdot (x-1) = x^2-1 \quad | \cdot 2$$

$$(x-4)(x+1) + (x+4)(x-1) = 2x^2-2$$

$$x^2-4x+x-4+x^2+4x-x-4-2x^2 = -2$$

$$0 = -2+4+4$$

$$0 \neq 6$$

$$K = \emptyset$$

Pr 10

Řešte v $\mathbb{Z}$ rovnice:

$$a) \frac{\frac{x}{3} - \frac{3x-4}{2}}{\frac{x}{2} + \frac{2x-5}{3}} = \frac{5}{4}$$

$$\frac{\frac{2x - 3(3x-4)}{6}}{\frac{3x + 2(2x-5)}{6}} = \frac{5}{4}$$

$$\frac{2x - 9x + 12}{6} \cdot \frac{6}{3x + 4x - 10} = \frac{5}{4}$$

$$\frac{-7x + 12}{7x - 10} = \frac{5}{4} \quad / \cdot 4(7x - 10) \quad x \neq \frac{10}{7}$$

$$4 \cdot (-7x + 12) = 5 \cdot (7x - 10)$$

$$-28x + 48 = 35x - 50$$

$$-28x - 35x = -50 - 48$$

$$-63x = -98 \quad / : (-63)$$

$$x = \frac{98}{63}$$

$$\underline{x = \frac{14}{9}} \notin \mathbb{Z}, \text{ tzn. } \boxed{K = \emptyset}$$

$$b) \frac{\frac{x}{3} - \frac{1}{12}}{\frac{x}{4} + \frac{1}{6}} = \frac{\frac{x}{21} - \frac{1}{4}}{\frac{x}{28} - \frac{1}{6}}$$

$$\frac{\frac{4x-1}{12}}{\frac{3x+2}{12}} = \frac{\frac{4x-21}{84}}{\frac{3x-14}{84}}$$

$$\frac{4x-1}{12} \cdot \frac{12}{3x+2} = \frac{4x-21}{84} \cdot \frac{84}{3x-14}$$

$$\frac{4x-1}{3x+2} = \frac{4x-21}{3x-14} \quad / \cdot (3x+2)(3x-14) \quad x \neq \frac{-2}{3} \quad x \neq \frac{14}{3}$$

$$(4x-1)(3x-14) = (4x-21)(3x+2)$$

$$12x^2 - 56x - 3x + 14 = 12x^2 + 8x - 63x - 42$$

$$-56x - 3x - 8x + 63x = -42 - 14$$

$$-4x = -56 \quad / : (-4)$$

$$\underline{x = 14}$$

$$\boxed{K = \{14\}}$$