KVADRATICKÉ ROVNICE

Př 1 V oboru $\mathbb{R}$ řešte rovnice:

a) $x^2 + 1{,}5x - 4{,}5 = 0$

$D = 1{,}5^2 - 4 \cdot (-4{,}5) =$

$= 2{,}25 + 18 = 20{,}25$

$x_{1,2} = \frac{-1{,}5 \pm \sqrt{20{,}25}}{2} = \frac{-1{,}5 \pm 4{,}5}{2}$

$x_1 = \frac{-1{,}5 + 4{,}5}{2} = \frac{3}{2}$

$x_2 = \frac{-1{,}5 - 4{,}5}{2} = \frac{-6}{2} = (-3)$

$K = \{-3; \frac{3}{2}\}$

b) $x^2 - 6x + 5 = 0$

$D = 36 - 4 \cdot 5 = 36 - 20 = 16$

$x_{1,2} = \frac{6 \pm \sqrt{16}}{2} = \frac{6 \pm 4}{2}$

$x_1 = \frac{6+4}{2} = \frac{10}{2} = 5$

$x_2 = \frac{6-4}{2} = \frac{2}{2} = 1$

$K = \{1; 5\}$

c) $x^2 + 0{,}9x - 0{,}36 = 0$

$D = 0{,}81 - 4 \cdot (-0{,}36) =$

$= 0{,}81 + 1{,}44 = 2{,}25$

$x_{1,2} = \frac{-0{,}9 \pm \sqrt{2{,}25}}{2} = \frac{-0{,}9 \pm 1{,}5}{2}$

$x_1 = \frac{-0{,}9 + 1{,}5}{2} = \frac{0{,}6}{2} = 0{,}3$

$x_2 = \frac{-0{,}9 - 1{,}5}{2} = \frac{-2{,}4}{2} = -1{,}2$

$K = \{-1{,}2; 0{,}3\}$

d) $2x^2 - 5x - 3 = 0$

$D = 25 - 4 \cdot 2 \cdot (-3) =$

$= 25 + 24 = 49$

$x_{1,2} = \frac{5 \pm \sqrt{49}}{4} = \frac{5 \pm 7}{4}$

$x_1 = \frac{5+7}{4} = \frac{12}{4} = 3$

$x_2 = \frac{5-7}{4} = \frac{-2}{4} = \frac{-1}{2}$

$K = \{\frac{-1}{2}; 3\}$

e) $x^2 + 7x = 0$

$x(x+7) = 0$

$x_1 = 0$ $\quad$ $x_2 = (-7)$

$K = \{-7; 0\}$

f) $x^2 + x + 1 = 0$

$D = 1 - 4 \cdot 1 = 1 - 4 = -3$

$D < 0$ $\quad$ $K = \emptyset$

g) $-x^2 - 6x + 91 = 0$

$D = 36 - 4 \cdot (-1) \cdot 91 = 36 + 364 = 400$

$x_{1,2} = \frac{6 \pm \sqrt{400}}{-2} = \frac{6 \pm 20}{-2}$

$x_1 = \frac{6+20}{-2} = \frac{26}{-2} = (-13)$

$x_2 = \frac{6-20}{-2} = \frac{-14}{-2} = 7$

$K = \{-13; 7\}$

h) $3x^2 + 23x - 70 = 0$

$D = 529 - 4 \cdot 3 \cdot (-70) =$

$= 529 + 840 = 1369$

$x_{1,2} = \frac{-23 \pm \sqrt{1369}}{6} = \frac{-23 \pm 37}{6}$

$x_1 = \frac{-23+37}{6} = \frac{14}{6} = \frac{7}{3}$

$x_2 = \frac{-23-37}{6} = \frac{-60}{6} = (-10)$

$K = \{-10; \frac{7}{3}\}$

i) $x^2 + 2x - 63 = 0$

$D = 4 - 4 \cdot (-63) = 4 + 252 = 256$

$x_{1,2} = \frac{-2 \pm \sqrt{256}}{2} = \frac{-2 \pm 16}{2}$

$x_1 = \frac{-2+16}{2} = \frac{14}{2} = 7$

$x_2 = \frac{-2-16}{2} = \frac{-18}{2} = (-9)$

$K = \{-9; 7\}$

j) $x^2 - 2\sqrt{2}x + 2 = 0$

$D = 4 \cdot 2 - 4 \cdot 2 = 8 - 8 = 0$

$x_{1,2} = \frac{2\sqrt{2} \pm \sqrt{0}}{2} = \frac{2\sqrt{2}}{2}$

$x = \sqrt{2}$

$K = \{\sqrt{2}\}$

k) $-x^2 - 20x + 156 = 0$

$D = 400 - 4 \cdot (-1) \cdot 156 = 400 + 624 = 1024$

$x_{1,2} = \frac{20 \pm \sqrt{1024}}{-2} = \frac{20 \pm 32}{-2}$

$x_1 = \frac{20+32}{-2} = \frac{52}{-2} = (-26)$

$x_2 = \frac{20-32}{-2} = \frac{-12}{-2} = 6$

$K = \{-26; 6\}$

l) $14x + 5x^2 = 0$

$5x^2 + 14x = 0$

$D = 196 - 4 \cdot 5 \cdot 0 = 196$

$x_{1,2} = \frac{-14 \pm \sqrt{196}}{10} = \frac{-14 \pm 14}{10}$

$x_1 = \frac{-14+14}{10} = \frac{0}{10} = 0$

$x_2 = \frac{-14-14}{10} = \frac{-28}{10} = \frac{-14}{5}$

$K = \{\frac{-14}{5}; 0\}$

m) $3x^2 + 5x + 1 = 0$

$D = 25 - 4 \cdot 3 \cdot 1 = 25 - 12 = 13$

$x_{1,2} = \frac{-5 \pm \sqrt{13}}{6}$

$x_1 = \frac{-5+\sqrt{13}}{6}$ $\quad x_2 = \frac{-5-\sqrt{13}}{6}$

$K = \{\frac{-5-\sqrt{13}}{6}; \frac{-5+\sqrt{13}}{6}\}$

n) $4x^2 - 4x - 1 = 0$

$D = 16 - 4 \cdot 4 \cdot (-1) = 16 + 16 = 32$

$x_{1,2} = \frac{4 \pm \sqrt{32}}{8} = \frac{4 \pm \sqrt{4 \cdot 8}}{8} = \frac{4 \pm \sqrt{16 \cdot 2}}{8} = \frac{4 \pm 4\sqrt{2}}{8}$

$x_1 = \frac{4+4\sqrt{2}}{8} = \frac{4(1+\sqrt{2})}{8_2} = \frac{1+\sqrt{2}}{2}$

$x_2 = \frac{4-4\sqrt{2}}{8} = \frac{4(1-\sqrt{2})}{8_2} = \frac{1-\sqrt{2}}{2}$

o) $2x^2 + 8x + 15 = 0$

$D = 64 - 4 \cdot 2 \cdot 15 = 64 - 120 = -56$

$D < 0$

$K = \emptyset$

$K = \{\frac{1-\sqrt{2}}{2}; \frac{1+\sqrt{2}}{2}\}$

p) $3x^2-5x=0$

$x(3x-5)=0$

$x_1=0$; $3x-5=0$, $3x=5$, $x_2=\frac{5}{3}$

$K=\{0;\frac{5}{3}\}$

r) $x^2+2{,}7x-10{,}9=0$

$D=7{,}29-4\cdot(-10{,}9)\cdot 1=$

$=7{,}29+43{,}6=50{,}89$

$x_{1,2}=\frac{-2{,}7\pm\sqrt{50{,}89}}{2}=\frac{-2{,}7\pm 7{,}1}{2}$

$x_1=\frac{-2{,}7+7{,}1}{2}=\frac{4{,}4}{2}=2{,}2$

$x_2=\frac{-2{,}7-7{,}1}{2}=\frac{-9{,}8}{2}=-4{,}9$

$K=\{-4{,}9;2{,}2\}$

s) $x^2-1{,}58x+2{,}86=0$

$D=2{,}4964-4\cdot 2{,}86=$

$=2{,}4964-11{,}44=-8{,}9436$

$D<0$

$K=\emptyset$

t) $-5x^2+9=0$

$D=0-4\cdot(-5)\cdot 9=180$

$x_{1,2}=\frac{0\pm\sqrt{180}}{-10}=\frac{\pm\sqrt{36\cdot 5}}{-10}=\frac{\pm 6\sqrt{5}}{-10}$

$x_1=\frac{-6\sqrt{5}}{-10}=\frac{3\sqrt{5}}{5}$; $x_2=\frac{6\sqrt{5}}{-10}=\frac{-3\sqrt{5}}{5}$

$K=\{\frac{-3\sqrt{5}}{5};\frac{3\sqrt{5}}{5}\}$

u) $5x^2-18x-8=0$

$D=324-4\cdot 5\cdot(-8)=$

$=324+160=484$

$x_{1,2}=\frac{18\pm\sqrt{484}}{10}=\frac{18\pm 22}{10}$

$x_1=\frac{18+22}{10}=\frac{40}{10}=4$; $x_2=\frac{18-22}{10}=\frac{-4}{10}=\frac{-2}{5}$

$K=\{\frac{-2}{5};4\}$

Př 2 V oboru R řešte rovnice:

a) $(x-4)(4x-3)+3=0$

$4x^2-3x-16x+12+3=0$

$4x^2-19x+15=0$

$D=361-4\cdot 4\cdot 15=361-240$

$D=121$

$x_{1,2}=\frac{19\pm\sqrt{121}}{8}=\frac{19\pm 11}{8}$

$x_1=\frac{19+11}{8}=\frac{30}{8}=\frac{15}{4}$

$x_2=\frac{19-11}{8}=\frac{8}{8}=1$

$K=\{1;\frac{15}{4}\}$

b) $(2x-1)(1+x)=x(1+x)$

$2x+2x^2-1-x=x+x^2$

$x^2-1=0$

$(x-1)(x+1)=0$

$x_1=1$; $x_2=(-1)$

$K=\{-1;1\}$

c) $(4x-3)^2-(6x+4)^2=69$

$16x^2-24x+9-(36x^2+48x+16)=69$

$16x^2-24x+9-36x^2-48x-16=69$

$-20x^2-72x-76=0 \quad /:(-4)$

$5x^2+18x+19=0$

$D=324-4\cdot5\cdot19=324-380$

$D=-56$

$D<0$ $\quad K=\emptyset$

d) $(x-3)^2+(x-4)^2=(x-2)^2$

$x^2-6x+9+x^2-8x+16=x^2-4x+4$

$x^2-10x+21=0$

$D=100-4\cdot21=100-84=16$

$x_{1,2}=\frac{10\pm\sqrt{16}}{2}=\frac{10\pm4}{2}$

$x_1=\frac{10+4}{2}=\frac{14}{2}=7$

$x_2=\frac{10-4}{2}=\frac{6}{2}=3$

$K=\{3;7\}$

e) $7x(x-3)=-2(x^2+5)$

$7x^2-21x=-2x^2-10$

$9x^2-21x+10=0$

$D=441-4\cdot9\cdot10=441-360$

$D=81$

$x_{1,2}=\frac{21\pm\sqrt{81}}{18}=\frac{21\pm9}{18}$

$x_1=\frac{21+9}{18}=\frac{30}{18}=\frac{5}{3}$

$x_2=\frac{21-9}{18}=\frac{12}{18}=\frac{2}{3}$

$K=\{\frac{2}{3};\frac{5}{3}\}$

f) $(x+3)(x-2)=(3x+2)(4x-3)$

$x^2-2x+3x-6=12x^2-9x+8x-6$

$-11x^2+2x=0$

$D=4-4\cdot(-11)\cdot0=4$

$x_{1,2}=\frac{-2\pm\sqrt{4}}{-22}=\frac{-2\pm2}{-22}$

$x_1=\frac{-2\pm2}{-22}=\frac{0}{-22}=0$

$x_2=\frac{-2-2}{-22}=\frac{-4}{-22}=\frac{2}{11}$

$K=\{0;\frac{2}{11}\}$

g) $(x+3)^2+(x+4)^2=(x+5)^2$

$x^2+6x+9+x^2+8x+16=x^2+10x+25$

$x^2+4x=0$

$x(x+4)=0$

$x_1=0$ $\quad x_2=(-4)$

$K=\{-4;0\}$

h) $(2x+1)(x-3)+(2x-1)(x+2)=4x-1$

$2x^2-6x+x-3+2x^2+4x-x-2=4x-1$

$4x^2-6x-4=0 \quad /:2$

$2x^2-3x-2=0$

$D=9-4\cdot2\cdot(-2)=9+16=25$

$x_{1,2}=\frac{3\pm\sqrt{25}}{4}=\frac{3\pm5}{4}$

$x_1=\frac{3+5}{4}=\frac{8}{4}=2$

$x_2=\frac{3-5}{4}=\frac{-2}{4}=\frac{-1}{2}$

$K=\{-\frac{1}{2};2\}$

Př 3 V oboru $\mathbb{R}$ řeště rovnice:

a) $2x-1+\frac{1}{2x+1}=2$

$2x+1\neq 0$
$2x\neq -1$
$x\neq -\frac{1}{2}$

$(2x-1)(2x+1)+1=2(2x+1)$
$4x^2+2x-2x-1+1=4x+2$
$4x^2-4x-2=0 \quad /:2$
$2x^2-2x-1=0$

$D=4-4\cdot 2\cdot(-1)=4+8=12$

$x_{1,2}=\frac{2\pm\sqrt{12}}{4}=\frac{2\pm\sqrt{4\cdot 3}}{4}=\frac{2\pm 2\sqrt{3}}{4}$

$x_1=\frac{2+2\sqrt{3}}{4}=\frac{2(1+\sqrt{3})}{4}=\frac{1+\sqrt{3}}{2}$

$x_2=\frac{2-2\sqrt{3}}{4}=\frac{2(1-\sqrt{3})}{4}=\frac{1-\sqrt{3}}{2}$

$K=\left\{\frac{1-\sqrt{3}}{2};\frac{1+\sqrt{3}}{2}\right\}$

b) $\frac{3}{8}-\frac{1}{x-4}=-\frac{1}{x+2}$

$x\neq 4$
$x\neq -2$

$3(x-4)(x+2)-8(x+2)=-8(x-4)$
$3(x^2-4x+2x-8)-8x-16=-8x+32$
$3x^2-12x+6x-24-8x-16=-8x+32$
$3x^2-6x-72=0 \quad /:3$
$x^2-2x-24=0$

$D=4-4\cdot(-24)=4+96=100$

$x_{1,2}=\frac{2\pm\sqrt{100}}{2}=\frac{2\pm 10}{2}$

$x_1=\frac{2+10}{2}=\frac{12}{2}=6$

$x_2=\frac{2-10}{2}=\frac{-8}{2}=(-4)$

$K=\{-4;6\}$

c) $\frac{x^2}{5}-\frac{2x}{3}=\frac{x+5}{6} \quad /\cdot 30$

$6x^2-20x=5(x+5)$
$6x^2-20x=5x+25$
$6x^2-25x-25=0$

$D=625-4\cdot 6\cdot(-25)$
$D=625+600=1225$

$x_{1,2}=\frac{25\pm\sqrt{1225}}{12}=\frac{25\pm 35}{12}$

$x_1=\frac{25+35}{12}=\frac{60}{12}=5$

$x_2=\frac{25-35}{12}=\frac{-10}{12}=\frac{-5}{6}$

$K=\left\{\frac{-5}{6};5\right\}$

d) $\frac{5}{x-2}+\frac{3}{x-3}=\frac{7}{x-1}$

$x\neq 1$
$x\neq 2$
$x\neq 3$

$5(x-3)(x-1)+3(x-2)(x-1)=7(x-2)(x-3)$
$5(x^2-3x-x+3)+3(x^2-2x-x+2)=7(x^2-3x-2x+6)$
$5x^2-15x-5x+15+3x^2-6x-3x+6=7x^2-21x-14x+42$
$x^2+6x-21=0$

$D=36-4\cdot(-21)=36+84=120$

$x_{1,2}=\frac{-6\pm\sqrt{120}}{2}=\frac{-6\pm\sqrt{4\cdot 30}}{2}=\frac{-6\pm 2\sqrt{30}}{2}$

$x_1=\frac{-6+2\sqrt{30}}{2}=\frac{2(-3+\sqrt{30})}{2}=-3+\sqrt{30}$

$x_2=\frac{-6-2\sqrt{30}}{2}=\frac{2(-3-\sqrt{30})}{2}=-3-\sqrt{30}$

$K=\{-3-\sqrt{30};-3+\sqrt{30}\}$

e) $\frac{5-3x}{3-5x} + \frac{3-5x}{5-3x} = \frac{5}{2}$ $/\cdot 2\cdot(3-5x)(5-3x)$ $x \neq \frac{5}{3}$ $x \neq \frac{3}{5}$

$2(5-3x)(5-3x) + 2(3-5x)(3-5x) = 5(3-5x)(5-3x)$

$2(25-30x+9x^2) + 2(9-30x+25x^2) = 5(15-9x-25x+15x^2)$

$50-60x+18x^2+18-60x+50x^2 = 75-45x-125x+75x^2$

$68x^2-75x^2-120x+170x+68-75=0$

$-7x^2+50x-7=0$

$D = 2500-4\cdot(-7)\cdot(-7) = 2500-196$

$D = 2304$

$x_{1,2} = \frac{-50\pm\sqrt{2304}}{-14} = \frac{-50\pm 48}{-14}$

$x_1 = \frac{-50+48}{-14} = \frac{-2}{-14} = \frac{1}{7}$ $x_2 = \frac{-50-48}{-14} = \frac{-98}{-14} = 7$

$K = \{\frac{1}{7}; 7\}$

f) $\frac{x+3}{x-3} + \frac{x-6}{x+6} = \frac{11}{5}$ $/\cdot 5(x-3)(x+6)$ $x \neq 3$ $x \neq -6$

$5(x+3)(x+6) + 5(x-6)(x-3) = 11(x-3)(x+6)$

$5(x^2+6x+3x+18) + 5(x^2-3x-6x+18) = 11(x^2+6x-3x-18)$

$5x^2+30x+15x+90+5x^2-15x-30x+90 = 11x^2+66x-33x-198$

$10x^2-11x^2-66x+33x+180+198=0$

$-x^2-33x+378=0$

$D = 1089-4\cdot(-1)\cdot 378$

$D = 1089+1512 = 2601$

$x_{1,2} = \frac{33\pm\sqrt{2601}}{-2} = \frac{33\pm 51}{-2}$

$x_1 = \frac{33+51}{-2} = \frac{84}{-2} = (-42)$ $x_2 = \frac{33-51}{-2} = \frac{-18}{-2} = 9$

$K = \{-42; 9\}$

g) $\frac{3x^2+8}{x-1} - \frac{2x-1}{4} = \frac{43+3x-2x^2}{4x-4}$ $/\cdot 4(x-1)$ $x \neq 1$

$4x-4 = 4(x-1)$

$4(3x^2+8) - (2x-1)(x-1) = 43+3x-2x^2$

$12x^2+32-(2x^2-2x-x+1) = 43+3x-2x^2$

$12x^2+32-2x^2+2x+x-1-43-3x+2x^2=0$

$12x^2-12=0$ $/:12$

$x^2-1=0$

$(x-1)(x+1)=0$

$x_1 = 1$ $x_2 = (-1)$

! podm. !

$K = \{-1\}$

h) $\frac{x}{x-2} = \frac{8}{(x-2)(x+2)} - \frac{5}{x+2}$ $/\cdot(x-2)(x+2)$

$x \neq \pm 2$

$x(x+2) = 8 - 5(x-2)$

$x^2 + 2x = 8 - 5x + 10$

$x^2 + 7x - 18 = 0$

$D = 49 - 4\cdot(-18) = 49 + 72 = 121$

$x_{1,2} = \frac{-7 \pm \sqrt{121}}{2} = \frac{-7 \pm 11}{2}$

$x_1 = \frac{-7+11}{2} = \frac{4}{2} = 2$ $x_2 = \frac{-7-11}{2} = \frac{-18}{2} = (-9)$

! podm. ! $K = \{-9\}$

i) $\frac{1}{y-3} - 1 = \frac{4}{y+3} - \frac{5}{3-y}$ $/\cdot(y-3)(y+3)$ $y \neq \pm 3$

$(y+3) - (y-3)(y+3) = 4(y-3) + 5(y+3)$

$y + 3 - (y^2 - 9) = 4y - 12 + 5y + 15$

$y + 3 - y^2 + 9 - 4y + 12 - 5y - 15 = 0$

$-y^2 - 8y + 9 = 0$

$D = 64 - 4\cdot(-1)\cdot 9 = 64 + 36 = 100$

$x_{1,2} = \frac{8 \pm \sqrt{100}}{-2} = \frac{8 \pm 10}{-2}$

$x_1 = \frac{8+10}{-2} = \frac{18}{-2} = (-9)$ $x_2 = \frac{8-10}{-2} = \frac{-2}{-2} = 1$

$K = \{-9; 1\}$