SOUSTAVY TŘÍ LINEÁRNÍCH ROVNIC O TŘECH NEZNÁMÝCH

Př 1)

$$\begin{aligned} 2x-3y+4z&=5\\ 3x+4y-2z&=0\\ -4x+2y+3z&=8 \end{aligned}$$

$3x = 2z-4y$

$x = \frac{2z-4y}{3}$

$2\cdot\frac{2z-4y}{3} - 3y + 4z = 5 \quad /\cdot 3$

$-4\cdot\frac{2z-4y}{3} + 2y + 3z = 8 \quad /\cdot 3$

$2(2z-4y) - 9y + 12z = 15$

$-4(2z-4y) + 6y + 9z = 24$

$4z - 8y - 9y + 12z = 15$

$-8z + 16y + 6y + 9z = 24$

$-17y + 16z = 15$

$22y + z = 24 \rightarrow z = 24-22y$

$-17y + 16(24-22y) = 15$

$-17y + 384 - 352y = 15$

$-369y = 15-384$

$-369y = -369$

$y = 1$

$x = \frac{2\cdot 2-4\cdot 1}{3}$

$x = \frac{4-4}{3}$

$x = \frac{0}{3}$

$x = 0$

$z = 24-22\cdot 1$

$z = 24-22$

$z = 2$

$K = \{[0;1;2]\}$

Př 2)

$$\begin{aligned} x-y-z&=5\\ y-x-z&=1\\ z-x-y&=-15 \end{aligned}$$

$y = 1+x+z$

$x-(1+x+z)-z=5$

$z-x-(1+x+z)=-15$

$x-1-x-z-z=5$

$z-x-1-x-z=-15$

$-2z = 5+1$

$-2x = -15+1$

$-2z = 6 \quad /:(-2)$

$-2x = -14 \quad /:(-2)$

$z = (-3)$

$x = 7$

$y = 1+7+(-3)$

$y = 1+7-3$

$y = 5$

$K = \{[7;5;-3]\}$

Př 3)

$$\begin{aligned} -3x+3y-4z&=1\\ x+5y+5z&=2\\ -x+2y-z&=1 \end{aligned}$$

$\rightarrow x = 2-5y-5z$

$-3(2-5y-5z)+3y-4z=1$

$-(2-5y-5z)+2y-z=1$

$-6+15y+15z+3y-4z=1$

$-2+5y+5z+2y-z=1$

$18y+11z=7 \quad /\cdot 4$

$7y+4z=3 \quad /\cdot(-11)$

$72y+44z=28$

$-77y-44z=-33$

$-5y=-5 \quad /:(-5)$

$y=1$

$7y+4z=3$

$7\cdot 1+4z=3$

$7+4z=3$

$4z=3-7$

$4z=-4$

$z=(-1)$

$x=2-5-5(-1)$

$x=2-5+5$

$x=2$

$K=\{[2;1;-1]\}$

Př 4)

$$\begin{aligned} x+2y+3z&=0\\ x-y+z&=0\\ x+y-2z&=0 \end{aligned}$$

$\Rightarrow x = y-z$

$y-z+2y+3z=0$

$y-z+y-2z=0$

$3y+2z=0 \quad /\cdot 3$

$2y-3z=0 \quad /\cdot 2$

$9y+6z=0$

$4y-6z=0$

$13y=0$

$y=0$

$3y+2z=0$

$3\cdot 0+2z=0$

$2z=0$

$z=0$

$x=0-0$

$x=0$

$K=\{[0;0;0]\}$

Př 5)

$$\begin{aligned} 2x+y&=7\\ y-3z&=-9\\ 5z-x&=18 \end{aligned}$$

$\rightarrow y = 3z-9$

$2x+3z-9=7$

$5z-x=18$

$2x+3z=16$

$-x+5z=18 \quad /\cdot 2$

$2x+3z=16$

$-2x+10z=36$

$13z=52 \quad /:13$

$z=4$

$2x+3z=16$

$2x+3\cdot 4=16$

$2x+12=16$

$2x=16-12$

$2x=4$

$x=2$

$y=3\cdot 4-9$

$y=12-9$

$y=3$

$K=\{[2;3;4]\}$

Př 6)

$$7x + 6y - 7z = 100$$
$$x - 2y + z = 0 \quad \rightarrow x = 2y - z$$
$$3x + y - 2z = 0$$

$$7(2y - z) + 6y - 7z = 100$$
$$3(2y - z) + y - 2z = 0$$

$$14y - 7z + 6y - 7z = 100$$
$$6y - 3z + y - 2z = 0$$

$$20y - 14z = 100 \quad /\cdot 7$$
$$7y - 5z = 0 \quad /\cdot(-20)$$

$$140y - 98z = 700$$
$$-140y + 100z = 0$$
$$2z = 700$$
$$z = 350$$

$$20y - 14z = 100$$
$$20y - 14 \cdot 350 = 100$$
$$20y - 4900 = 100$$
$$20y = 5000$$
$$y = 250$$

$$x = 2y - z$$
$$x = 2 \cdot 250 - 350$$
$$x = 500 - 350$$
$$x = 150$$

$$K = \{[150; 250; 350]\}$$

Př 7)

$$\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1 \quad /\cdot 12$$
$$\frac{x}{3} + \frac{y}{4} + \frac{z}{5} = 1 \quad /\cdot 60$$
$$\frac{x}{4} + \frac{y}{5} + \frac{z}{6} = 1 \quad /\cdot 60$$

$$6x + 4y + 3z = 12 \quad \rightarrow 3z = 12 - 6x - 4y$$
$$20x + 15y + 12z = 60 \qquad z = \frac{12 - 6x - 4y}{3}$$
$$15x + 12y + 10z = 60$$

$$20x + 15y + 12 \cdot \frac{12 - 6x - 4y}{3} = 60$$
$$15x + 12y + 10 \cdot \frac{12 - 6x - 4y}{3} = 60 \quad /\cdot 3$$

$$20x + 15y + 4(12 - 6x - 4y) = 60$$
$$45x + 36y + 10(12 - 6x - 4y) = 180$$

$$20x + 15y + 48 - 24x - 16y = 60$$
$$45x + 36y + 120 - 60x - 40y = 180$$

$$-4x - y = 12 \quad /\cdot(-4)$$
$$-15x - 4y = 60$$

$$16x + 4y = -48$$
$$-15x - 4y = 60$$
$$x = 12$$

$$z = \frac{12 - 6 \cdot 12 - 4 \cdot (-60)}{3}$$
$$z = \frac{12 - 72 + 240}{3}$$
$$z = \frac{180}{3}$$
$$z = 60$$

$$-4x - y = 12$$
$$-4 \cdot 12 - y = 12$$
$$-48 - y = 12$$
$$-y = 12 + 48$$
$$-y = 60$$
$$y = (-60)$$

$$K = \{[12; -60; 60]\}$$

Př 8)

$$2x - \frac{y+z}{7} = \frac{11}{12} \quad /\cdot 84$$

$$3y - \frac{x+z}{9} = \frac{11}{12} \quad /\cdot 36$$

$$4z - \frac{x+y}{2} = \frac{7}{12} \quad /\cdot 12$$

$$168x - 12(y+z) = 77$$
$$108y - 4(x+z) = 33$$
$$48z - 6(x+y) = 7$$

$$168x - 12y - 12z = 77$$
$$108y - 4x - 4z = 33 \longrightarrow -4x = 33 + 4z - 108y$$
$$48z - 6x - 6y = 7 \qquad 4x = -33 - 4z + 108y$$
$$x = \frac{-33 - 4z + 108y}{4}$$

$$168 \cdot \frac{-33 - 4z + 108y}{4} - 12y - 12z = 77$$

$$48z - 6 \cdot \frac{-33 - 4z + 108y}{4} - 6y = 7 \quad /\cdot 2$$

$$42 \cdot (-33 - 4z + 108y) - 12y - 12z = 77$$
$$2 \cdot 48z - 3 \cdot (-33 - 4z + 108y) - 12y = 14$$

$$-1386 - 168z + 4536y - 12y - 12z = 77$$
$$96z + 99 + 12z - 324y - 12y = 14$$

$$4524y - 180z = 1463 \quad /\cdot 3$$
$$-336y + 108z = -85 \quad /\cdot 5$$

$$13572y - 540z = 4389$$
$$-1680y + 540z = -425$$

$$11892y = 3964$$
$$y = \frac{3964}{11892}$$
$$y = \frac{1}{3}$$

180 = 2 · 90 = 2 · 9 · 10 = 2 · 3 · 3 · 2 · 5

108 = 2 · 54 = 2 · 6 · 9 = 2 · 2 · 3 · 3 · 3

$n(180,108) = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 540$

$$-336y + 108z = -85$$
$$-336 \cdot \frac{1}{3} + 108z = -85$$
$$-112 + 108z = -85$$
$$108z = 112 - 85$$
$$108z = 27$$
$$z = \frac{27}{108}$$
$$z = \frac{1}{4}$$

$$x = \frac{-33 - 4z + 108y}{4} \quad /\cdot 4$$
$$4x = -33 - 4 \cdot \frac{1}{4} + 108 \cdot \frac{1}{3}$$
$$4x = -33 - 1 + 36$$
$$4x = 2$$
$$x = \frac{1}{2}$$

$$K = \left\{\left[\frac{1}{2}; \frac{1}{3}; \frac{1}{4}\right]\right\}$$

Př 9)

$$\frac{3x+y}{z+1}=2 \qquad /\cdot(z+1) \qquad z\neq -1$$

$$\frac{3y+z}{x+1}=2 \qquad /\cdot(x+1) \qquad x\neq -1$$

$$\frac{3z+x}{y+1}=2 \qquad /\cdot(y+1) \qquad y\neq -1$$

$$3x+y=2(z+1)$$
$$3y+z=2(x+1)$$
$$3z+x=2(y+1)$$

$$3x+y=2z+2$$
$$3y+z=2x+2$$
$$3z+x=2y+2$$

$$3x+y-2z=2 \quad \Rightarrow \quad y=2+2z-3x$$
$$-2x+3y+z=2$$
$$x-2y+3z=2$$

$$-2x+3(2+2z-3x)+z=2$$
$$x-2(2+2z-3x)+3z=2$$

$$-2x+6+6z-9x+z=2$$
$$x-4-4z+6x+3z=2$$

$$-11x+7z=-4$$
$$7x-z=6 \qquad /\cdot 7$$

$$-11x+7z=-4$$
$$49x-7z=42$$

$$38x=38$$
$$x=1$$

$$7x-z=6$$
$$7-z=6$$
$$-z=6-7$$
$$-z=-1$$
$$z=1$$

$$y=2+2-3$$
$$y=1$$

$$K=\{[1;1;1]\}$$

Př 10)

$$x + 2y + 3z - 2t = 6 \quad \rightarrow x = 6 - 2y - 3z + 2t$$
$$2x - y - 2z - 3t = 8$$
$$3x + 2y - z + 2t = 4$$
$$2x - 3y + 2z + t = -8$$

$$2(6 - 2y - 3z + 2t) - y - 2z - 3t = 8$$
$$3(6 - 2y - 3z + 2t) + 2y - z + 2t = 4$$
$$2(6 - 2y - 3z + 2t) - 3y + 2z + t = -8$$

$$12 - 4y - 6z + 4t - y - 2z - 3t = 8$$
$$18 - 6y - 9z + 6t + 2y - z + 2t = 4$$
$$12 - 4y - 6z + 4t - 3y + 2z + t = -8$$

$$-5y - 8z + t = -4 \quad \rightarrow t = 5y + 8z - 4$$
$$-4y - 10z + 8t = -14$$
$$-7y - 4z + 5t = -20$$

$$-4y - 10z + 8(5y + 8z - 4) = -14$$
$$-7y - 4z + 5(5y + 8z - 4) = -20$$

$$-4y - 10z + 40y + 64z - 32 = -14$$
$$-7y - 4z + 25y + 40z - 20 = -20$$

$$36y + 54z = 18$$
$$18y + 36z = 0 \quad /\cdot(-2)$$

$$36y + 54z = 18$$
$$-36y - 72z = 0$$

$$-18z = 18 \quad /:(-18)$$
$$z = (-1)$$

$$18y + 36z = 0$$
$$18y - 36 = 0$$
$$18y = 36 \quad /:18$$
$$y = 2$$

$$t = 5y + 8z - 4$$
$$t = 5 \cdot 2 + 8 \cdot (-1) - 4$$
$$t = 10 - 8 - 4$$
$$t = (-2)$$

$$x = 6 - 2y - 3z + 2t$$
$$x = 6 - 2 \cdot 2 - 3 \cdot (-1) + 2 \cdot (-2)$$
$$x = 6 - 4 + 3 - 4$$
$$x = 1$$

$$K = \{[1; 2; -1; -2]\}$$
$x \; y \; z \; t$