

SOUSTAVY DVOU LINEÁRNÍCH ROVNIC O DVOU NEZNÁMÝCH

Pr 1

a) $4x + 3y = 6$

$2x + y = 4$

→

$y = 4 - 2x$

dosazovací metoda

$4x + 3(4 - 2x) = 6$

$4x + 12 - 6x = 6$

$4x - 6x = 6 - 12$

$-2x = -6$

$x = 3$

$y = 4 - 2 \cdot 3$

$y = 4 - 6$

$y = -2$

$K = \{[3, -2]\}$

b) $3x + 5y = 14$

$6x - 10y = 17$

sčítací metoda

$3x + 5y = 14 \quad | \cdot 2$

$6x - 10y = 17$

$6x + 10y = 28$

$6x - 10y = 17$

$12x = 45$

$x = \frac{45}{12}$

$x = \frac{15}{4}$

$K = \{[\frac{15}{4}, \frac{11}{20}]\}$

$3x + 5y = 14$

$3 \cdot \frac{15}{4} + 5y = 14$

$\frac{45}{4} + 5y = 14 \quad | \cdot 4$

$45 + 20y = 56$

$20y = 56 - 45$

$20y = 11$

$y = \frac{11}{20}$

c) $x + 3y = 20$

$x - 5y = 12$

srovnávací metoda

→ $x = 20 - 3y$

→ $x = 12 + 5y$

$20 - 3y = 12 + 5y$

$(x = x)$

$-3y - 5y = 12 - 20$

$-8y = -8$

$y = 1$

$x = 20 - 3 \cdot 1$

$x = 20 - 3$

$x = 17$

$K = \{[17, 1]\}$

d) $2x + y = 5$

$2x - y = 3$

grafická metoda

$2x + y = 5$

• $f_1: y = 5 - 2x$

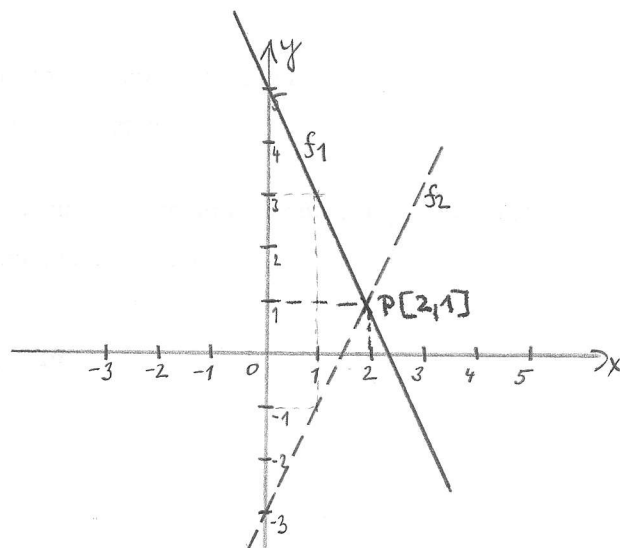
$2x - y = 3$

• $f_2: y = 2x - 3$

x	0	1
y	5	3

x	0	1
y	-3	-1

$K = \{[2, 1]\}$



$$\begin{aligned} e) \quad & 2x - 3y = 8 \\ & 3x - 2y = 27 \end{aligned}$$

sčítací metoda

$$\begin{aligned} 2x - 3y &= 8 & / \cdot (-3) \\ 3x - 2y &= 27 & / \cdot 2 \\ \hline -6x + 9y &= -24 \\ 6x - 4y &= 54 \\ \hline 5y &= 30 \\ y &= 6 \end{aligned}$$

$$\begin{aligned} 2x - 3y &= 8 \\ 2x - 3 \cdot 6 &= 8 \\ 2x - 18 &= 8 \\ 2x &= 8 + 18 \\ 2x &= 26 \\ x &= 13 \end{aligned}$$

$$K = \{[13; 6]\}$$

$$\begin{aligned} f) \quad & 2x + 3y = 1 \\ & 3x + 2y = 9 \end{aligned}$$

dosazovací metoda

$$\rightarrow 2y = 9 - 3x \\ y = \frac{9 - 3x}{2}$$

$$2x + 3 \cdot \frac{9 - 3x}{2} = 1 \quad / \cdot 2$$

$$4x + 3(9 - 3x) = 2$$

$$4x + 27 - 9x = 2$$

$$4x - 9x = 2 - 27$$

$$-5x = -25 \quad / \cdot (-5)$$

$$x = 5$$

$$y = \frac{9 - 3 \cdot 5}{2}$$

$$y = \frac{9 - 15}{2}$$

$$y = \frac{-6}{2} = -3$$

$$K = \{[5; -3]\}$$

$$\begin{aligned} g) \quad & 3x - 2y = 1 \\ & 4x - y + 2 = 0 \end{aligned}$$

grafická metoda

$$3x - 2y = 1$$

$$-2y = 1 - 3x$$

$$y = \frac{1 - 3x}{-2}$$

$$f_1: y = \frac{3x - 1}{2}$$

x	0	2
y	$-\frac{1}{2}$	$\frac{5}{2}$

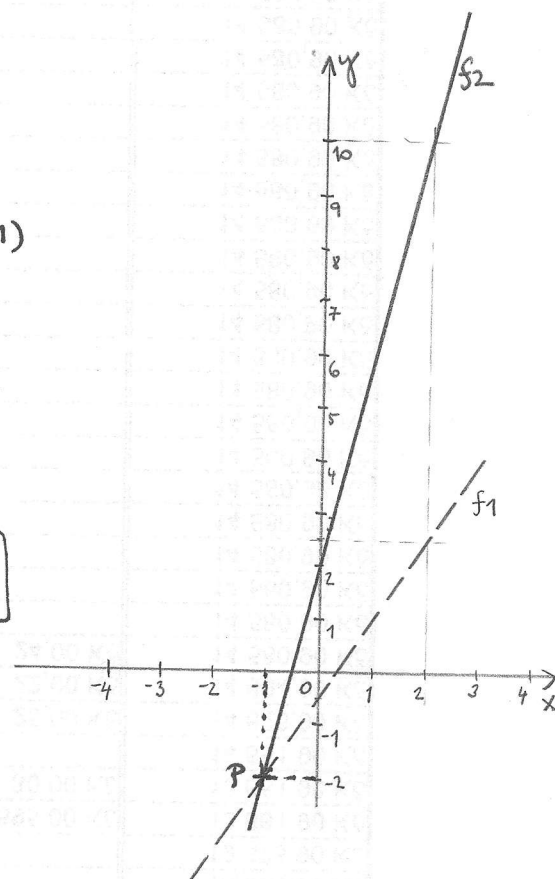
$$4x - y + 2 = 0$$

$$-y = -2 - 4x \quad / \cdot (-1)$$

$$f_2: y = 4x + 2$$

x	0	2
y	2	10

$$K = \{[-1; -2]\}$$



$$\begin{aligned} h) \quad & 5x + 2y = 23 \\ & 3x - y = 5 \end{aligned}$$

srovnávací metoda

$$\rightarrow 2y = 23 - 5x$$

$$y = \frac{23 - 5x}{2}$$

$$\rightarrow -y = 5 - 3x$$

$$y = -5 + 3x$$

$$\frac{23 - 5x}{2} = -5 + 3x \quad / \cdot 2$$

$$23 - 5x = -10 + 6x$$

$$-5x - 6x = -10 - 23$$

$$-11x = -33$$

$$x = 3$$

$$y = -5 + 3 \cdot 3$$

$$y = -5 + 9$$

$$y = 4$$

$$K = \{[3; 4]\}$$

Př2

a) $2(x+y) - 5(y-x) = 17$
 $3(x+2y) + 7(3x+5y) = 7$

$$\begin{array}{r} 2x + 2y - 5y + 5x = 17 \\ 3x + 6y + 21x + 35y = 7 \end{array}$$

$$\begin{array}{r} 7x - 3y = 17 \\ 24x + 41y = 7 \end{array} \rightarrow \begin{array}{l} -3y = 17 - 7x \quad / : (-3) \\ y = \frac{7x - 17}{3} \end{array}$$

$$24x + 41 \frac{7x - 17}{3} = 7 \quad / \cdot 3$$

$$72x + 41(7x - 17) = 21$$

$$72x + 287x - 697 = 21$$

$$72x + 287x = 21 + 697$$

$$359x = 718$$

$$\underline{\underline{x = 2}}$$

$$y = \frac{7 \cdot 2 - 17}{3}$$

$$y = \frac{14 - 17}{3}$$

$$y = \frac{-3}{3}$$

$$\underline{\underline{y = -1}}$$

$$K = \{[2; -1]\}$$

b) $(x+4)(y-2) = (x-5)(y+4)$
 $(x+6)(y-1) = (x-1)(y+2)$

$$xy - 2x + 4y - 8 = xy + 4x - 5y - 20$$

$$xy - x + 6y - 6 = xy + 2x - y - 2$$

$$-2x + 4y - 4x + 5y = -20 + 8$$

$$-x - 2x + 6y + y = -2 + 6$$

$$-6x + 9y = -12$$

$$-3x + 7y = 4 \quad / \cdot (-2)$$

$$-6x + 9y = -12$$

$$6x - 14y = -8$$

$$-5y = -20 \quad / : (-5)$$

$$\underline{\underline{y = 4}}$$

$$-3x + 7 \cdot 4 = 4$$

$$-3x + 28 = 4$$

$$-3x = 4 - 28$$

$$-3x = -24 \quad / : (-3)$$

$$\underline{\underline{x = 8}}$$

$$K = \{[8; 4]\}$$

c) $(x+5)(y-2) = (x+2)(y-1)$
 $(x-4)(y+7) = (x-3)(y+4)$

$$xy - 2x + 5y - 10 = xy - x + 2y - 2$$

$$xy + 7x - 4y - 28 = xy + 4x - 3y - 12$$

$$-2x + x + 5y - 2y = -2 + 10$$

$$7x - 4x - 4y + 3y = -12 + 28$$

$$-x + 3y = 8$$

$$3x - y = 16 \quad / \cdot 3$$

$$-x + 3y = 8$$

$$9x - 3y = 48$$

$$8x = 56$$

$$\underline{\underline{x = 7}}$$

$$-x + 3y = 8$$

$$-7 + 3y = 8$$

$$3y = 8 + 7$$

$$3y = 15$$

$$\underline{\underline{y = 5}}$$

$$K = \{[7; 5]\}$$

$$d) \quad 5(x-3) - 3(y+2) = 23$$

$$3(x-3) + 5(y+2) = 7$$

$$5x - 15 - 3y - 6 = 23$$

$$3x - 9 + 5y + 10 = 7$$

$$5x - 3y = 23 + 15 + 6$$

$$3x + 5y = 7 + 9 - 10$$

$$5x - 3y = 44 \quad / \cdot 5$$

$$3x + 5y = 6 \quad / \cdot 3$$

$$25x - 15y = 220$$

$$9x + 15y = 18$$

$$34x = 238 \quad / : 34$$

$$x = 7$$

$$5x - 3y = 44$$

$$5 \cdot 7 - 3y = 44$$

$$35 - 3y = 44$$

$$-3y = 44 - 35$$

$$-3y = 9$$

$$y = (-3)$$

$$K = \{[7; -3]\}$$

Pr 3

$$a) \quad \frac{x+y}{5} + \frac{y}{5} = -2 \quad / \cdot 5$$

$$\frac{2x-y}{3} - \frac{3x}{4} = \frac{3}{2} \quad / \cdot 12$$

$$x+y+y = -10$$

$$4(2x-y) - 3 \cdot 3x = 3 \cdot 6$$

$$x+y+y = -10$$

$$8x - 4y - 9x = 18$$

$$x + 2y = -10$$

$$-x - 4y = 18$$

$$-2y = 8$$

$$y = (-4)$$

$$x + 2y = -10$$

$$x + 2 \cdot (-4) = -10$$

$$x - 8 = -10$$

$$x = -10 + 8$$

$$x = (-2)$$

$$K = \{[-2; -4]\}$$

$$b) \quad \frac{2x+1}{5} - \frac{3y+2}{7} = 2y-x \quad / \cdot 35$$

$$\frac{3x-1}{4} + \frac{7y+2}{6} = 2x-y \quad / \cdot 12$$

$$7(2x+1) - 5(3y+2) = 35(2y-x)$$

$$3(3x-1) + 2(7y+2) = 12(2x-y)$$

$$14x + 7 - 15y - 10 = 70y - 35x$$

$$9x - 3 + 14y + 4 = 24x - 12y$$

$$14x + 35x - 15y - 70y = 10 - 7$$

$$9x - 24x + 14y + 12y = 3 - 4$$

$$49x - 85y = 3 \quad / \cdot 15$$

$$-15x + 26y = -1 \quad / \cdot 49$$

$$735x - 1275y = 45$$

$$-735x + 1274y = -49$$

$$49x - 85 \cdot 4 = 3$$

$$49x - 340 = 3$$

$$49x = 3 + 340$$

$$49x = 343 \quad / : 49$$

$$x = 7$$

$$K = \{[7; 4]\}$$

$$-y = -4$$

$$y = 4$$

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$$c) \frac{3x-2y}{5} + \frac{5x-3y}{3} = x+1 \quad | \cdot 15$$

$$\frac{2x-3y}{3} + \frac{4x-3y}{3} = y \quad | \cdot 3$$

$$3(3x-2y) + 5(5x-3y) = 15(x+1)$$

$$2x-3y + 4x-3y = 3y$$

$$9x-6y + 25x-15y = 15x+15$$

$$2x-3y + 4x-3y = 3y$$

$$9x+25x-15x-6y-15y = 15$$

$$2x+4x-3y-3y-3y = 0$$

$$19x-21y = 15 \quad | \cdot 3$$

$$6x-9y = 0 \quad | \cdot (-7)$$

$$57x-63y = 45$$

$$-42x+63y = 0$$

$$15x = 45$$

$$x = 3$$

$$19x-21y = 15$$

$$19 \cdot 3 - 21y = 15$$

$$57 - 21y = 15$$

$$-21y = 15 - 57$$

$$-21y = -42 \quad | : (-21)$$

$$y = 2$$

$$K = \{[3; 2]\}$$

$$d) \frac{2x-y+3}{3} - \frac{x-2y+3}{4} = 4 \quad | \cdot 12$$

$$\frac{3x-4y+3}{4} + \frac{4x-2y-9}{3} = 4 \quad | \cdot 12$$

$$4(2x-y+3) - 3(x-2y+3) = 4 \cdot 12$$

$$3(3x-4y+3) + 4(4x-2y-9) = 4 \cdot 12$$

$$8x-4y+12-3x+6y-9 = 48$$

$$9x-12y+9+16x-8y-36 = 48$$

$$8x-3x-4y+6y = 48+9-12$$

$$9x+16x-12y-8y = 48+36-9$$

$$5x+2y = 45 \quad | \cdot 10$$

$$25x-20y = 75$$

$$50x+20y = 450$$

$$25x-20y = 75$$

$$75x = 525$$

$$x = 7$$

$$K = \{[7; 5]\}$$

$$5x+2y = 45$$

$$5 \cdot 7 + 2y = 45$$

$$35 + 2y = 45$$

$$2y = 45 - 35$$

$$2y = 10$$

$$y = 5$$

Pr 4

$$\begin{aligned} a) \quad & 3(2x+3y) + 2(2x-3y) = 43 \\ & 8(2x+3y) - 3(2x-3y) = 73 \end{aligned}$$

$$\begin{aligned} 6x + 9y + 4x - 6y &= 43 \\ 16x + 24y - 6x + 9y &= 73 \end{aligned}$$

$$10x + 3y = 43 \quad | \cdot (-1)$$

$$10x + 33y = 73$$

$$-10x - 3y = -43$$

$$10x + 33y = 73$$

$$30y = 30$$

$$\underline{\underline{y = 1}}$$

$$10x + 3y = 43$$

$$10x + 3 = 43$$

$$10x = 43 - 3$$

$$10x = 40$$

$$\underline{\underline{x = 4}}$$

$$K = \{[4; 1]\}$$

$$b) \quad y = -\frac{1}{3}x + 2 \quad | \cdot 3$$

$$\frac{y}{2} + \frac{x}{6} = 1 \quad | \cdot 6$$

$$3y = -x + 6$$

$$3y + x = 6 \quad | \cdot (-1)$$

$$3y + x = 6$$

$$-3y - x = (-6)$$

$$\underline{\underline{0 = 0}}$$

nekonečně mnoho řešení, ale:

$$3y + x = 6$$

$$3y = 6 - x$$

$$\underline{\underline{y = \frac{6-x}{3} !}}$$

$$K = \{[x; \frac{6-x}{3}]\}$$

$$\begin{aligned} c) \quad & 2x + y = 3 \\ & y + 2x + 2 = 0 \end{aligned}$$

$$2x + y = 3$$

$$2x + y = -2$$

$$\underline{\underline{0 \neq 1}}$$

není řešení

$$K = \emptyset$$

$$\begin{aligned} d) \quad & 0,1x + 0,3y = 0,1 \quad | \cdot 10 \\ & 0,3x - 0,2y = -0,8 \quad | \cdot 10 \end{aligned}$$

$$x + 3y = 1 \quad | \cdot 2$$

$$3x - 2y = -8 \quad | \cdot 3$$

$$2x + 6y = 2$$

$$9x - 6y = -24$$

$$11x = -22$$

$$\underline{\underline{x = (-2)}}$$

$$x + 3y = 1$$

$$-2 + 3y = 1$$

$$3y = 1 + 2$$

$$3y = 3$$

$$\underline{\underline{y = 1}}$$

$$K = \{[-2; 1]\}$$