LINEÁRNÍ NEROVNICE O JEDNÉ NEZNÁMÉ

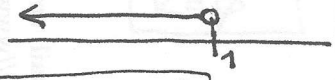
Př 1

a) $x+1 < -3x+5$

$x+3x < 5-1$

$4x < 4 \quad /:4$

$x < 1$

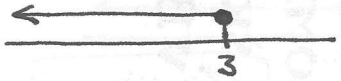


$K = (-\infty; 1)$

b) $5x+3 \leqq 6+4x$

$5x-4x \leqq 6-3$

$x \leqq 3$



$K = (-\infty; 3\rangle$

c) $5x+5 \geqslant 6x-1$

$5x-6x \geqslant -1-5$

$-x \geqslant -6 \quad /\cdot(-1)$

$x \leqq 6$

$K = (-\infty; 6\rangle$

d) $9-x-6x \geqslant 3-7x$

$-x-6x+7x \geqslant 3-9$

$0 \geqslant -6$

pravdivý zápis

$K = \mathbb{R}$

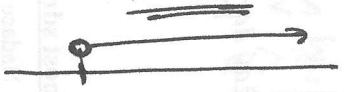
e) $6x+1 > 2(x-5)-1$

$6x+1 > 2x-10-1$

$6x-2x > -10-1-1$

$4x > -12 \quad /:4$

$x > (-3)$



$K = (-3; \infty)$

f) $x+5+x < 1+2x$

$x+x-2x < 1-5$

$0 < -4$

nepravdivý zápis

$K = \emptyset$

g) $6-4x-(2x-3)\cdot 5 \geqslant 0$

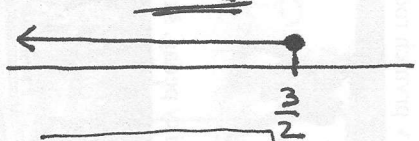
$6-4x-10x+15 \geqslant 0$

$-4x-10x \geqslant -6-15$

$-14x \geqslant -21 \quad /:(-14)$

$x \leqq \frac{21}{14}$

$x \leqq \frac{3}{2}$



$K = (-\infty; \frac{3}{2}\rangle$

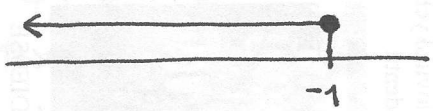
h) $2x+9 \leqq (4x-1)\cdot(-1)-2x$

$2x+9 \leqq -4x+1-2x$

$2x+4x+2x \leqq 1-9$

$8x \leqq -8 \quad /:8$

$x \leqq (-1)$



$K = (-\infty; -1\rangle$

Př 2

a) $x+3(x+4)<2+4(x+1)$

$x+3x+12<2+4x+4$

$x+3x-4x<2+4-12$

$0<-6$

nepravdivý zápis

$K=\emptyset$

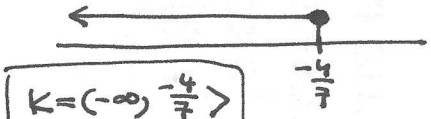
b) $7-(4x-1)\geq 3(x+4)$

$7-4x+1\geq 3x+12$

$-4x-3x\geq 12-7-1$

$-7x\geq 4 \quad /:(-7)$

$x\leq \frac{-4}{7}$



$K=(-\infty;\frac{-4}{7}\rangle$

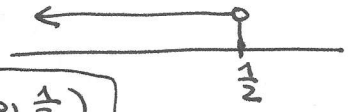
c) $(6-x)\cdot 3-5>5x-13+22$

$18-3x-5>5x-13+22$

$-3x-5x>-13+22-18+5$

$-8x>-4 \quad /:(-8)$

$x<\frac{1}{2}$



$K=(-\infty;\frac{1}{2})$

d) $5x-8(2-3x)<x+12$

$5x-16+24x<x+12$

$5x+24x-x<12+16$

$28x<28$

$x<1$

$K=(-\infty;1)$

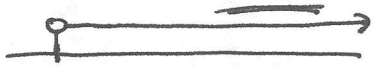
e) $5(x-1)-x(7-x)<x^2$

$5x-5-7x+x^2<x^2$

$5x-7x+\cancel{x^2}-\cancel{x^2}<5$

$-2x<5 \quad /:(-2)$

$x>\frac{-5}{2}$



$K=(\frac{-5}{2};\infty)$

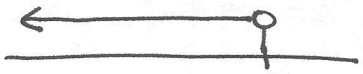
f) $(4x-1)^2+3x<(8x+1)(2x-4)$

$16x^2-8x+1+3x<16x^2-32x+2x-4$

$\cancel{16x^2}-8x+3x-\cancel{16x^2}+32x-2x<-4-1$

$25x<-5 \quad /:25$

$x<\frac{-1}{5}$



$K=(-\infty;\frac{-1}{5})$

Př 3

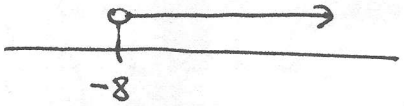
a) $\frac{x}{2}-1<7+x-4 \quad /\cdot 2$

$x-2<14+2x-8$

$x-2x<14-8+2$

$-x<8 \quad /\cdot(-1)$

$x>-8$



$K=(-8;\infty)$

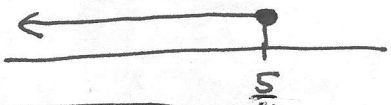
b) $\frac{3}{4}-x\geq\frac{1}{8}+\frac{x}{2} \quad /\cdot 8$

$6-8x\geq 1+4x$

$-8x-4x\geq 1-6$

$-12x\geq -5 \quad /:(-12)$

$x\leq\frac{5}{12}$



$K=(-\infty;\frac{5}{12}\rangle$

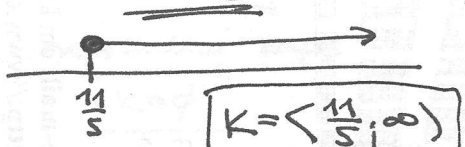
c) $\frac{x}{3}+\frac{1}{6} \leqq \frac{x}{2}-\frac{1}{5}$ /·30

$10x+5 \leqq 15x-6$

$10x-15x \leqq -6-5$

$-5x \leqq -11$ /:(-5)

$x \geqslant \frac{11}{5}$



$K=\langle \frac{11}{5}; \infty)$

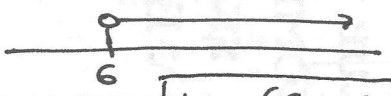
d) $5 < \frac{x}{2}-\frac{x}{3}+4$ /·6

$30 < 3x-2x+24$

$-3x+2x < 24-30$

$-x < -6$ /·(-1)

$x > 6$



$K=(6; \infty)$

Př 4

a) $\frac{x}{2}+1-\frac{2-x}{4} \leqq 0$ /·4

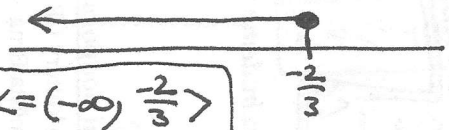
$2x+4-(2-x) \leqq 0$

$2x+4-2+x \leqq 0$

$2x+x \leqq -4+2$

$3x \leqq -2$ /:3

$x \leqq \frac{-2}{3}$



$K=(-\infty; \frac{-2}{3}\rangle$

b) $4 > \frac{x}{6}+\frac{3x-5}{2}$ /·6

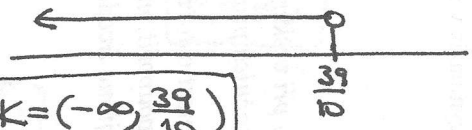
$24 > x+3(3x-5)$

$24 > x+9x-15$

$-x-9x > -15-24$

$-10x > -39$ /:(-10)

$x < \frac{39}{10}$



$K=(-\infty; \frac{39}{10})$

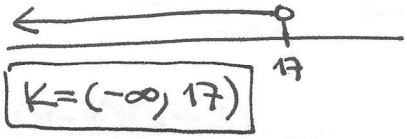
c) $\frac{2x-1}{3} < \frac{x+5}{2}$ /·6

$2(2x-1) < 3(x+5)$

$4x-2 < 3x+15$

$4x-3x < 15+2$

$x < 17$



$K=(-\infty; 17)$

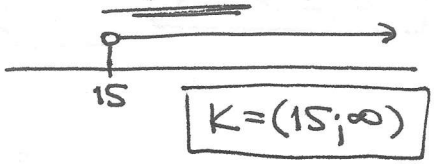
d) $\frac{5(x-1)}{6}-1 > \frac{2(x+1)}{3}$ /·6

$5(x-1)-6 > 4(x+1)$

$5x-5-6 > 4x+4$

$5x-4x > 4+5+6$

$x > 15$



$K=(15; \infty)$

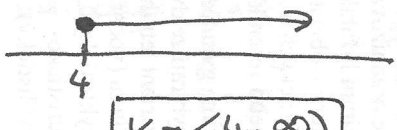
e) $\frac{x}{2}-\frac{x+2}{3} \geqslant 0$ /·6

$3x-2(x+2) \geqslant 0$

$3x-2x-4 \geqslant 0$

$3x-2x \geqslant 4$

$x \geqslant 4$



$K=\langle 4; \infty)$

f) $\frac{4x}{5} < 1-\frac{x-4}{5}$ /·5

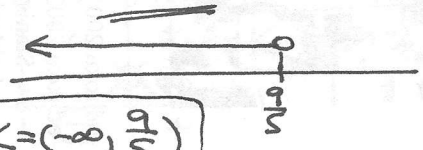
$4x < 5-(x-4)$

$4x < 5-x+4$

$4x+x < 5+4$

$5x < 9$ /:5

$x < \frac{9}{5}$



$K=(-\infty; \frac{9}{5})$

Př 5 a) $\frac{5x-1}{2} < \frac{10x-7}{3} - \frac{5x+1}{6}$ /·6

$3(5x-1) < 2(10x-7) - (5x+1)$

$15x - 3 < 20x - 14 - 5x - 1$

$15x - 20x + 5x < -14 - 1 + 3$

$0 < -12$

nepravdivý zápis

$K = \emptyset$

b) $\frac{3-2x}{5} + 8 \geqslant \frac{5x+2}{2} - x$ /·10

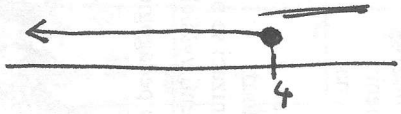
$2(3-2x) + 80 \geqslant 5(5x+2) - 10x$

$6 - 4x + 80 \geqslant 25x + 10 - 10x$

$-4x - 25x + 10x \geqslant 10 - 80 - 6$

$-19x \geqslant -76$ /:(-19)

$x \leqslant 4$



$K = (-\infty; 4\rangle$

c) $\frac{2x-1}{2} + \frac{x+1}{4} \leqslant \frac{7x+2}{3} - \frac{x}{6}$ /·12

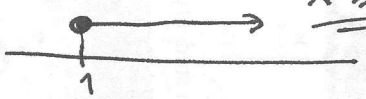
$6(2x-1) + 3(x+1) \leqslant 4(7x+2) - 2x$

$12x - 6 + 3x + 3 \leqslant 28x + 8 - 2x$

$12x + 3x - 28x + 2x \leqslant 8 + 6 - 3$

$-11x \leqslant 11$ /:(-11)

$x \geqslant (-1)$



$K = \langle -1; \infty)$

d) $\frac{2x-3}{5} + \frac{3x-4}{6} < \frac{9x-5}{10}$ /·30

$6(2x-3) + 5(3x-4) < 3(9x-5)$

$12x - 18 + 15x - 20 < 27x - 15$

$12x + 15x - 27x < -15 + 18 + 20$

$0 < 11$

pravdivý zápis

$K = \mathbb{R}$

Př 6 a) $\frac{x-5}{x-1} > 0$

nul. body: $x = 5$, $x = 1$

		1		5	
$x-5$	−		−	0	+
$x-1$	−	0	+		+
	⊕	‖	−	‖	⊕

$K = (-\infty; 1) \cup (5; \infty)$

b) $\frac{3-2x}{2x-5} \geqslant 0$

nul. body: $x = \frac{3}{2}$, $x = \frac{5}{2}$

		$\frac{3}{2}$		$\frac{5}{2}$	
$3-2x$	+	0	−		−
$2x-5$	−		−	0	+
	−	‖	⊕	‖	−

$K = \langle \frac{3}{2}; \frac{5}{2})$

c) $\frac{(x-4)(x+3)}{x-1} < 0$

nul. body: $x = 4$, $x = (-3)$, $x = 1$

		-3		1		4	
$x-4$	−		−		−	0	+
$x+3$	−	0	+		+		+
$x-1$	−		−	0	+		+
	⊖	‖	+	‖	⊖	‖	+

$K = (-\infty; -3) \cup (1; 4)$

d) $\frac{(2x-1)(x+2)}{x-5} \leqslant 0$

nul. body: $x = \frac{1}{2}$, $x = -2$, $x = 5$

		-2		$\frac{1}{2}$		5	
$2x-1$	−		−	0	+		+
$x+2$	−	0	+		+		+
$x-5$	−		−		−	0	+
	⊖	‖	+	‖	⊖	‖	+

$K = (-\infty; -2\rangle \cup \langle \frac{1}{2}; 5)$

Př 7 a) $\frac{2x+1}{x+2} > 1$

$\frac{2x+1}{x+2} - 1 > 0$

$\frac{2x+1-x-2}{x+2} > 0$

$\frac{x-1}{x+2} > 0$

nul. body:
$x = 1$
$x = (-2)$

	-2		1	
$x-1$	$-$	$-$	$\varnothing$	$+$
$x+2$	$-$	$\varnothing$	$+$	$+$
	$\oplus$	$-$	$\varnothing$	$\oplus$

$K = (-\infty; -2) \cup (1; \infty)$

b) $\frac{3x-1}{x+3} \leqq 2$

$\frac{3x-1}{x+3} - 2 \leqq 0$

$\frac{3x-1-2x-6}{x+3} \leqq 0$

$\frac{x-7}{x+3} \leqq 0$

nul. body:
$x = 7$
$x = (-3)$

	-3		7	
$x-7$	$-$	$-$	0	$+$
$x+3$	$-$	$\varnothing$	$+$	$+$
	$+$	$\ominus$	0	$+$

$K = (-3; 7\rangle$

c) $\frac{3x+2}{1-x} \leqq 3$

$\frac{3x+2}{1-x} - 3 \leqq 0$

$\frac{3x+2-3+3x}{1-x} \leqq 0$

$\frac{6x-1}{1-x} \leqq 0$

nul. body:
$x = \frac{1}{6}$
$x = 1$

	$\frac{1}{6}$		1	
$6x-1$	$-$	0	$+$	$+$
$1-x$	$+$	$+$	$\varnothing$	$-$
	$\ominus$	$+$	$\varnothing$	$\ominus$

$K = (-\infty; \frac{1}{6}\rangle \cup (1; \infty)$

d) $\frac{x-1}{2-x} > 2$

$\frac{x-1}{2-x} - 2 > 0$

$\frac{x-1-4+2x}{2-x} > 0$

$\frac{3x-5}{2-x} > 0$

nul. body:
$x = \frac{5}{3}$
$x = 2$

	$\frac{5}{3}$		2	
$3x-5$	$-$	$\varnothing$	$+$	$+$
$2-x$	$+$	$+$	$\varnothing$	$-$
	$-$	$\oplus$	$\varnothing$	$-$

$K = (\frac{5}{3}; 2)$

Př 8 a) $\frac{x+1}{x+2} - \frac{4-x}{1-x} \leqq 0$

$\frac{(x+1)(1-x) - (4-x)(x+2)}{(x+2)(1-x)} \leqq 0$

$\frac{x - x^2 + 1 - x - (4x + 8 - x^2 - 2x)}{(x+2)(1-x)} \leqq 0$

$\frac{x - x^2 + 1 - x - 4x - 8 + x^2 + 2x}{(x+2)(1-x)} \leqq 0$

$\frac{-2x - 7}{(x+2)(1-x)} \leqq 0$

nul. body: $x = \frac{-7}{2}$
$x = -2$
$x = 1$

	$-\frac{7}{2}$		-2		1	
$-2x-7$	$+$	0	$-$	$-$	$-$	$-$
$x+2$	$-$	$-$	$\varnothing$	$+$	$+$	$+$
$1-x$	$+$	$+$	$+$	$+$	$\varnothing$	$-$
	$\ominus$	$+$	$\varnothing$	$\ominus$	$\varnothing$	$+$

$K = (-\infty; -\frac{7}{2}\rangle \cup (-2; 1)$

b) $\frac{4}{x+3} \geqq \frac{4}{x-3}$

$\frac{4}{x+3} - \frac{4}{x-3} \geqq 0$

$\frac{4(x-3)-4(x+3)}{(x+3)(x-3)} \geqq 0$

$\frac{4x-12-4x-12}{(x+3)(x-3)} \geqq 0$

$\frac{-24}{(x+3)(x-3)} \geqq 0 \quad |\cdot(-1)$ (pozn. násobíme -1, abychom měli v čitateli kladnou hodnotu)

$\frac{24}{(x+3)(x-3)} \leqq 0$

nul. body: $x=-3$, $x=3$

	-3		3	
$x+3$	$-$	$+$		$+$
$x-3$	$-$		$+$	$+$
	$+$	$-$	$+$	

$K=(-3;3)$

c) $\frac{2}{1-x} \leqq \frac{-2}{x+1}$

$\frac{2}{1-x} + \frac{2}{x+1} \leqq 0$

$\frac{2(x+1)+2(1-x)}{(1-x)(x+1)} \leqq 0$

$\frac{2x+2+2-2x}{(1-x)(x+1)} \leqq 0$

$\frac{4}{(1-x)(x+1)} \leqq 0$

nul. body: $x=-1$, $x=1$

	-1		1	
$1-x$	$+$	$+$		$-$
$x+1$	$-$	$+$	$+$	
	$-$	$+$	$-$	

$K=(-\infty;-1) \cup (1;\infty)$

d) $\frac{5-x}{x-1} + \frac{1+4x}{2x+2} < 1$

$\frac{5-x}{x-1} + \frac{1+4x}{2x+2} - 1 < 0$

$\frac{(5-x)(2x+2)+(1+4x)(x-1)-(x-1)(2x+2)}{(x-1)(2x+2)} < 0$

$\frac{10x+10-2x^2-2x+x-1+4x^2-4x-(2x^2+2x-2x-2)}{(x-1)(2x+2)} < 0$

$\frac{10x+10-2x^2-2x+x-1+4x^2-4x-2x^2-2x+2x+2}{(x-1)(2x+2)} < 0$

$\frac{5x+11}{(x-1)(2x+2)} < 0$

nul. body: $x=\frac{-11}{5}$, $x=1$, $x=-1$

	$\frac{-11}{5}$		-1		1	
$5x+11$	$-$	$+$		$+$		$+$
$x-1$	$-$		$-$		$+$	$+$
$2x+2$	$-$		$-$	$+$		$+$
	$-$	$+$		$-$	$+$	

$K=(-\infty;\frac{-11}{5}) \cup (-1;1)$