Něco málo k přípravě na 4. písemnou práci (PMP3)

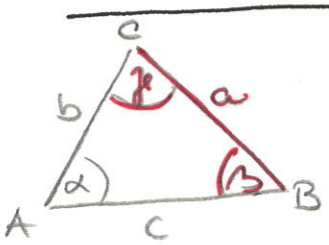
Př 1

a) $a = 165$
$\beta = 40°50'$
$\gamma = 69°20'$
$\alpha = ?$
$b = ?$
$c = ?$
$S = ?$

$\alpha = 180° - (\beta + \gamma) = 180° - (40°50' + 69°20') =$
$= 180° - 110°10' = \underline{69°50'}$

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$ $\quad b = \frac{a \cdot \sin\beta}{\sin\alpha} = \frac{165 \cdot \sin 40°50'}{\sin 69°50'} =$

$= \frac{165 \cdot 0{,}6538}{0{,}9386} = 114{,}93 \doteq \underline{115}$



$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$ $\quad c = \frac{a \cdot \sin\gamma}{\sin\alpha} = \frac{165 \cdot \sin 69°20'}{\sin 69°50'} =$

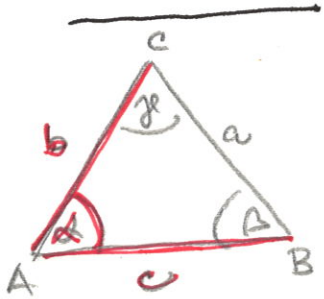
$= \frac{165 \cdot 0{,}9356}{0{,}9386} = 164{,}47 \doteq \underline{164{,}5}$

$S = \frac{1}{2} ab \sin\gamma = \frac{1}{2} \cdot 165 \cdot 115 \cdot \sin 69°20' = \frac{1}{2} \cdot 165 \cdot 115 \cdot 0{,}9356$

$S \doteq \underline{8876{,}5}$

b) $b = 32$
$c = 40$
$\alpha = 100°21'$
$a = ?$
$\beta = ?$
$\gamma = ?$
$S = ?$

$a^2 = b^2 + c^2 - 2bc\cos\alpha$
$a^2 = 32^2 + 40^2 - 2 \cdot 32 \cdot 40 \cdot \cos 100°21'$
$a^2 = 1024 + 1600 - 2560 \cdot (-0{,}1796)$
$a^2 = 2624 + 459{,}776 = 3083{,}776$
$a \doteq \underline{55{,}53}$



$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$

$\sin\beta = \frac{b \cdot \sin\alpha}{a} = \frac{32 \cdot \sin 100°21'}{55{,}53} = \frac{32 \cdot 0{,}9837}{55{,}53}$

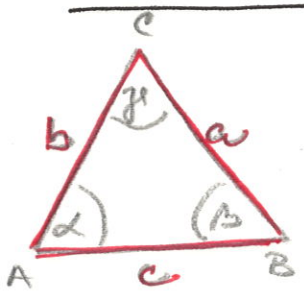
$\sin\beta = 0{,}5668$ $\quad \beta = \underline{34°31'}$

$\gamma = 180° - (\alpha + \beta) = 180° - (100°21' + 34°31') =$
$= 180° - 134°52' = \underline{45°08'}$

$S = \frac{1}{2} bc \sin\alpha = \frac{1}{2} \cdot 32 \cdot 40 \cdot \sin 100°21' =$

$= \frac{1}{2} \cdot 32 \cdot 40 \cdot 0{,}9837 \doteq \underline{629{,}568}$

c) $a = 4{,}2$
$b = 3{,}8$
$c = 5{,}5$
$\alpha = ?$
$\beta = ?$
$\gamma = ?$
$S = ?$



$c^2 = a^2 + b^2 - 2ab\cos\gamma$

$5{,}5^2 = 4{,}2^2 + 3{,}8^2 - 2 \cdot 4{,}2 \cdot 3{,}8 \cdot \cos\gamma$

$30{,}25 = 17{,}64 + 14{,}44 - 31{,}92\cos\gamma$

$31{,}92\cos\gamma = 1{,}83 \quad / : 31{,}92$

$\cos\gamma = 0{,}0573$

$\gamma = 86°42'$

$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$ $\quad \sin\alpha = \frac{a \cdot \sin\gamma}{c} = \frac{4{,}2 \cdot \sin 86°42'}{5{,}5}$

$\sin\alpha = \frac{4{,}2 \cdot 0{,}9983}{5{,}5} = 0{,}7623$

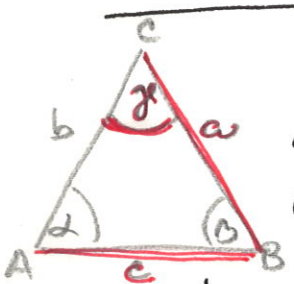
$\alpha = 49°40'$

$\beta = 180° - (\alpha + \gamma) = 180° - (49°40' + 86°42') =$
$= 180° - 136°22' = 43°38'$

$S = \frac{1}{2} ab \sin\gamma = \frac{1}{2} \cdot 4{,}2 \cdot 3{,}8 \cdot \sin 86°42' = \frac{1}{2} \cdot 4{,}2 \cdot 3{,}8 \cdot 0{,}9983$

$S = 7{,}97$

d) $a = 5{,}2$
$c = 8{,}8$
$\gamma = 52°08'$
$b = ?$
$\alpha = ?$
$\beta = ?$
$S = ?$



$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$

$\frac{5{,}2}{\sin\alpha} = \frac{8{,}8}{\sin 52°08'}$

$\sin\alpha = \frac{5{,}2 \cdot \sin 52°08'}{8{,}8} = \frac{5{,}2 \cdot 0{,}7894}{8{,}8} = 0{,}4664$

$\alpha = 27°48'$

$\beta = 180° - (\alpha + \gamma) = 180° - (27°48' + 52°8') = 180° - 79°56'$

$\beta = 100°4'$

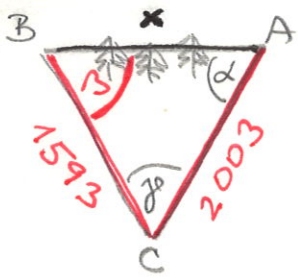
$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$ $\quad b = \frac{c \cdot \sin\beta}{\sin\gamma} = \frac{8{,}8 \cdot \sin 100°4'}{\sin 52°8'} = \frac{8{,}8 \cdot 0{,}9846}{0{,}7894}$

$b \doteq 10{,}98$

$S = \frac{1}{2} ab \sin\gamma = \frac{1}{2} \cdot 5{,}2 \cdot 10{,}98 \cdot \sin 52°8' = \frac{1}{2} \cdot 5{,}2 \cdot 10{,}98 \cdot 0{,}7894$

$S \doteq 22{,}54$

Př 2



$$\frac{2003}{\sin 63°23'} = \frac{1593}{\sin\alpha}$$

$$\sin\alpha = \frac{1593 \cdot \sin 63°23'}{2003} = \frac{1593 \cdot 0{,}8940}{2003}$$

$$\sin\alpha = 0{,}7110 \qquad \alpha = 45°18'$$

$$\gamma = 180° - (\alpha+\beta) = 180° - (45°18' + 63°23') =$$

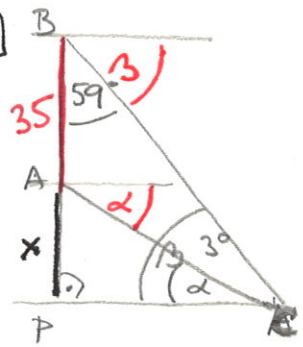
$$= 180° - 108°41' = 71°19'$$

$$\frac{x}{\sin\gamma} = \frac{b}{\sin\beta} \qquad x = \frac{2003 \cdot \sin 71°19'}{\sin 63°23'} = \frac{2003 \cdot 0{,}9473}{0{,}8940} \doteq 2122{,}4\ \text{m}$$

Délka projektované cesty z obce A do obce B je přibližně 2122 m.

(pozn. šlo by počítat i pomocí kosinovy věty)

Př 3



$|AC| = m$

$$\frac{m}{\sin 59°} = \frac{35}{\sin 3°}$$

$$m = \frac{35 \cdot \sin 59°}{\sin 3°} = \frac{35 \cdot 0{,}8571}{0{,}0523}$$

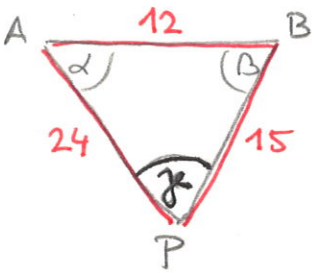
$$m \doteq 573{,}59$$

$$\frac{x}{m} = \sin 28° \quad \text{(pravoúhlý } \Delta\text{)}$$

$$x = 573{,}59 \cdot \sin 28° = 573{,}59 \cdot 0{,}4694 \doteq 264{,}24\ \text{m}$$

Vrchol kopce je 264,24 m nad rovinou pozorovacího místa.

Př 4



$$12^2 = 24^2 + 15^2 - 2 \cdot 24 \cdot 15 \cdot \cos\gamma$$

$$144 = 576 + 225 - 720 \cdot \cos\gamma$$

$$720\cos\gamma = 657 \quad /:720$$

$$\cos\gamma = 0{,}9125$$

$$\gamma = 24°8'$$

Předmět se jeví pozorovateli v zorném úhlu 24°8'

Př 5

a) $a_n = 6 - n$

$a_1 = 6 - 1 = \underline{5}$
$a_2 = 6 - 2 = \underline{4}$
$a_3 = 6 - 3 = \underline{3}$
$a_4 = 6 - 4 = \underline{2}$
$a_5 = 6 - 5 = \underline{1}$

b) $a_n = n(2n-1)$

$a_1 = 1 \cdot (2-1) = \underline{1}$
$a_2 = 2 \cdot (4-1) = 2 \cdot 3 = \underline{6}$
$a_3 = 3 \cdot (6-1) = 3 \cdot 5 = \underline{15}$
$a_4 = 4 \cdot (8-1) = 4 \cdot 7 = \underline{28}$
$a_5 = 5 \cdot (10-1) = 5 \cdot 9 = \underline{45}$

Př 6

a) $x, 2x, 3x, 4x, \ldots$

$\underline{a_n = n \cdot x}$

b) $a, a^2, a^3, a^4, \ldots$

$\underline{a_n = a^n}$

Př 7

a) $a_1 = 3$, $a_n = a_{n-1} - 2$

$a_2 = a_1 - 2 = 3 - 2 = \underline{1}$
$a_3 = a_2 - 2 = 1 - 2 = \underline{(-1)}$
$a_4 = a_3 - 2 = -1 - 2 = \underline{(-3)}$
$a_5 = a_4 - 2 = -3 - 2 = \underline{(-5)}$

b) $a_1 = 1$, $a_2 = 2$, $a_{n+1} = a_n - a_{n-1}$

$a_3 = a_2 - a_1 = 2 - 1 = \underline{1}$
$a_4 = a_3 - a_2 = 1 - 2 = \underline{(-1)}$
$a_5 = a_4 - a_3 = -1 - 1 = \underline{(-2)}$

c) $a_1 = 3$, $a_{n+1} = 4a_n$

$a_2 = 4a_1 = 4 \cdot 3 = \underline{12}$
$a_3 = 4a_2 = 4 \cdot 12 = \underline{48}$
$a_4 = 4a_3 = 4 \cdot 48 = \underline{192}$
$a_5 = 4a_4 = 4 \cdot 192 = \underline{768}$

d) $a_1 = 4$, $a_2 = 3$, $a_{n+2} = \dfrac{a_{n+1}}{a_n}$

$a_3 = \dfrac{a_2}{a_1} = \underline{\dfrac{3}{4}}$

$a_4 = \dfrac{a_3}{a_2} = \dfrac{\frac{3}{4}}{3} = \dfrac{3}{4} : 3 = \dfrac{\cancel{3}^{1}}{4} \cdot \dfrac{1}{\cancel{3}_{1}} = \underline{\dfrac{1}{4}}$

$a_5 = \dfrac{a_4}{a_3} = \dfrac{\frac{1}{4}}{\frac{3}{4}} = \dfrac{1}{\cancel{4}_{1}} \cdot \dfrac{\cancel{4}^{1}}{3} = \underline{\dfrac{1}{3}}$

Př 8

a) $a_1 = 5$, $d = 4$

$a_2 = a_1 + d = 5 + 4 = \underline{9}$
$a_3 = a_1 + 2d = 5 + 2 \cdot 4 = 5 + 8 = \underline{13}$
$a_4 = a_1 + 3d = 5 + 3 \cdot 4 = 5 + 12 = \underline{17}$
$a_5 = a_1 + 4d = 5 + 4 \cdot 4 = 5 + 16 = \underline{21}$

b) $a_1 = 0$, $d = -2$

$a_2 = a_1 + d = 0 + (-2) = \underline{(-2)}$
$a_3 = a_1 + 2d = 0 + 2 \cdot (-2) = \underline{(-4)}$
$a_4 = a_1 + 3d = 0 + 3 \cdot (-2) = \underline{(-6)}$
$a_5 = a_1 + 4d = 0 + 4 \cdot (-2) = \underline{(-8)}$

Př 9

a) $a_1 = 450$, $d = -24$, $a_n = 210$, $n = ?$, $s_n = ?$

$a_n = a_1 + (n-1)d$
$210 = 450 + (n-1) \cdot (-24)$
$210 = 450 - 24n + 24$
$24n = 450 + 24 - 210$
$24n = 264$
$\underline{n = 11}$

$s_n = \dfrac{n}{2}(a_1 + a_n)$
$s_n = \dfrac{11}{2}(450 + 210)$
$s_n = \dfrac{11}{2} \cdot 660$
$s_n = 11 \cdot 330 = \underline{3630}$

$n = 11$
$s_n = 3630$

b) $s_n = 245,\ d = 5,\ a_n = 47,\ n = ?,\ a_1 = ?$

$s_n = \frac{n}{2}(a_1 + a_n)$

$245 = \frac{n}{2}(a_1 + 47)$

$245 = \frac{n}{2}(52 - 5n + 47)$

$245 = \frac{n}{2}(99 - 5n) \quad |\cdot 2$

$490 = n(99 - 5n)$

$490 = 99n - 5n^2$

$5n^2 - 99n + 490 = 0$

$a_n = a_1 + (n-1)d$

$47 = a_1 + (n-1)\cdot 5$

$-a_1 = -47 + 5n - 5$

$-a_1 = -52 + 5n \quad |\cdot(-1)$

$a_1 = 52 - 5n$

$n_{1,2} = \frac{99 \pm \sqrt{9801 - 9800}}{10} = \frac{99 \pm \sqrt{1}}{10} = \frac{99 \pm 1}{10}$

$n_1 = \frac{100}{10} = \underline{10}$ $\quad n_2 = \frac{98}{10} = \underline{9{,}8}$

n_2 nevyhovuje, nebot $n_2 \notin \mathbb{N}$ (n... označuje kolikátý člen posloupnosti to je a proto $n \in \mathbb{N}$!)

$a_1 = 52 - 5\cdot 10$

$a_1 = 52 - 50 = \underline{2}$

$a_1 = 2$
$n = 10$

Př 10

$a_2 + a_4 = 22$
$a_3 + a_6 = 31$

$a_1 = ?$
$d = ?$

$a_2 + a_4 = 22$

$(a_1 + d) + (a_1 + 3d) = 22$

$2a_1 + 4d = 22$

$2a_1 = 22 - 4d$

$a_3 + a_6 = 31$

$(a_1 + 2d) + (a_1 + 5d) = 31$

$2a_1 + 7d = 31$

$(22 - 4d) + 7d = 31$

$7d - 4d = 31 - 22$

$3d = 9$

$\underline{d = 3}$

$2a_1 = 22 - 4d$

$2a_1 = 22 - 12$

$2a_1 = 10$

$\underline{a_1 = 5}$

$a_1 = 5$
$d = 3$

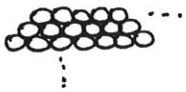
Př 11

$n = 10$ (počet řad)

$a_1 = 15$ (vrchní řada)

$d = 1$ (přírůstek dřívek)

$s_n = ?$ (součet všech dřívek)



$s_n = \frac{n}{2}(a_1 + a_n)$

$s_{10} = \frac{10}{2}(a_1 + a_{10})$

$s_{10} = 5\cdot(15 + a_{10})$

$s_{10} = 5\cdot(15 + 24)$

$s_{10} = 5\cdot 39$

$\underline{s_{10} = 195}$

$a_n = a_1 + (n-1)d$

$a_{10} = a_1 + 9d$

$a_{10} = 15 + 9\cdot 1 = 15 + 9$

$\underline{a_{10} = 24}$

Označených dřívek je celkem 195.

Př 12

$S_n = 1224$ (celk. počet diváků v kině)
$a_1 = 40$ (sedadel v první řadě)
$d = 4$ (přírůstek sedadel v každé další řadě)
$n = ?$ (počet řad sedadel) (pozn. $n \in \mathbb{N}$)

$S_n = \frac{n}{2}(a_1 + a_n)$

$a_n = a_1 + (n-1)d$
$a_n = 40 + (n-1)\cdot 4$
$a_n = 40 + 4n - 4$
$a_n = 36 + 4n$

$1224 = \frac{n}{2}(40 + 36 + 4n)$
$2448 = n(76 + 4n)$
$2448 = 76n + 4n^2$
$0 = 4n^2 + 76n - 2448 \quad |:4$
$0 = n^2 + 19n - 612$

$n_{1,2} = \frac{-19 \pm \sqrt{361 + 2448}}{2}$

$n_{1,2} = \frac{-19 \pm \sqrt{2809}}{2} = \frac{-19 \pm 53}{2}$

$n_1 = \frac{-19+53}{2} = 17$ $\quad n_2 = \frac{-19-53}{2} = \frac{-72}{2} = -36$ nevyhovuje, $n_2 \notin \mathbb{N}$

Počet řad v kině je 17.

Př 13

a) $a_1 = 2, \; q = \frac{1}{3}$

$a_2 = a_1 \cdot q = 2 \cdot \frac{1}{3} = \frac{2}{3}$
$a_3 = a_1 \cdot q^2 = 2\left(\frac{1}{3}\right)^2 = 2 \cdot \frac{1}{9} = \frac{2}{9}$
$a_4 = a_1 \cdot q^3 = 2 \cdot \left(\frac{1}{3}\right)^3 = 2 \cdot \frac{1}{27} = \frac{2}{27}$
$a_5 = a_1 \cdot q^4 = 2 \cdot \left(\frac{1}{3}\right)^4 = 2 \cdot \frac{1}{81} = \frac{2}{81}$

b) $a_1 = 8, \; q = \frac{1}{2}$

$a_2 = 8 \cdot \frac{1}{2} = \frac{8}{2} = 4$
$a_3 = a_1 \cdot q^2 = 8\left(\frac{1}{2}\right)^2 = 8 \cdot \frac{1}{4} = 2$
$a_4 = a_1 \cdot q^3 = 8 \cdot \left(\frac{1}{2}\right)^3 = 8 \cdot \frac{1}{8} = 1$
$a_5 = a_1 \cdot q^4 = 8 \cdot \left(\frac{1}{2}\right)^4 = 8 \cdot \frac{1}{16} = \frac{1}{2}$

Př 14

a) $a_1 = 18, \; q = 3, \; n = 7, \; a_n = ?, \; S_n = ?$

$a_n = a_1 \cdot q^{n-1}$
$a_7 = 18 \cdot 3^6$
$a_7 = 18 \cdot 729$
$a_7 = 13122$

$S_n = a_1 \cdot \frac{q^n - 1}{q - 1}$ (nebo $q \neq 1$)

$S_7 = 18 \cdot \frac{3^7 - 1}{3 - 1} = 18 \cdot \frac{2187 - 1}{2} = 18 \cdot \frac{2186}{2}$

$S_7 = 19674$

b) $a_1 = 12, \; q = 1, \; n = 5, \; a_n = ?, \; S_n = ?$

$a_n = a_1 \cdot q^{n-1}$
$a_5 = 12 \cdot 1^4 = 12 \cdot 1$
$a_5 = 12$

$S_n = n \cdot a_1$ (nebo $q = 1$)

$S_5 = 5 \cdot 12$
$S_5 = 60$